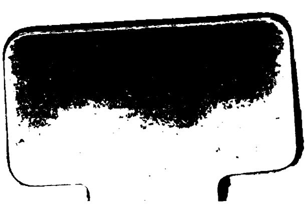

AN ELEMENTARY TREATISE

ON

GEOMETRICAL DRAWING,

WITH NUMEROUS EXAMPLES,

Chiefly Selected from Army Examination Papers.

BY THE

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LONDON:
RELFE BROS., 6, CHARTERHOUSE BUILDINGS,
ALDERSGATE STREET.

1880.

PREFACE.

THIS Manual is intended primarily for the use of Students who are preparing for the Preliminary Examination for entrance to the Royal Military Academy at Woolwich, or to the Royal Military College at Sandhurst. In either case Geometrical Drawing is one of the compulsory subjects.

But, apart from this special object, the Author's experience in tuition has convinced him that the subject deserves a place in the curriculum of studies in any good school. Exact drawing greatly facilitates the conception of geometrical methods, and frequently arouses, in the mind of a young student, an interest which merely scientific reasoning from rough diagrams fails to awaken.

Edgeborough, Guildford,
September 1st, 1880.

AN ELEMENTARY TREATISE

ON

GEOMETRICAL DRAWING.

SCALES.

ABBREVIATIONS.—The double accent (″) denotes inches; ABC denotes the *angle* ABC; $\triangle$ ABC denotes the triangle ABC.

1. **Diagonal scales** are used to measure distances too minute to be otherwise found.

In Fig. i. we have a diagonal scale to read *inches*, *tenths of an inch*, and *hundredths of an inch*, or, as it is generally expressed, "inches to two places of decimals."

In this figure, the distance AB, for example, $= 3.64''$.

$$\begin{aligned} \text{For DB} &= 3'' \\ \text{AC} &= \tfrac{6}{10}'' \\ \text{and CD} &= \tfrac{4}{100}'' \end{aligned}$$

The general rule for taking off such distances is this. Measure from the intersection of the *vertical* 3 and the *horizontal* 4, to the point where this horizontal line is cut by the *diagonal through* 6.

Before proceeding further the student is recommended to draw lines, say 3.27", 5.63", 3.09", 5.8", 3.46".

They should be taken off the *half-inch diagonal scale* given on the Protractor. These distances being *doubled* will be the lengths required.

2. *General method of constructing* **any** *diagonal scale, to read, say, A, B, C.*

(i.) Draw as many equidistant horizontal lines as will make the same number of spaces as there are C in B.

(ii.) Mark off vertical divisions each of which shall represent A.

(iii.) Subdivide (at the top) the first vertical division into as many equal parts as there are B in A.

(iv.) Draw the parallel diagonals.

Thus suppose A, B, C, to be **yards, feet,** and **inches,** and the scale to be one inch to the yard.

(i.) We make 12 equal spaces; (because there are 12" in 1 foot.)

(ii.) We draw verticals one inch apart; (because a yard is to be represented on the scale by 1".)

(iii.) We subdivide the first division into 3 equal parts; (because there are 3 ft. in one yard.)

(iv.) We draw the parallel diagonals.

This scale is drawn in Fig. ii.

To take off any length, say 4 yds. 2 ft. 7 in. from the scale, we must measure from the intersection of the vertical

4, and the horizontal 7, to the point where this horizontal line is cut by the diagonal through 2. (AB, in Fig. ii.)

3. All these lines, horizontal, vertical, and diagonal, may be drawn by the Marquois Scales. By moving the index on the triangle successively over the same number of graduations on the scale, we can rule lines which are not only parallel, but *equidistant* also. Further, this distance may be *specified*. Say that our parallels are required to be $\frac{1}{10}$'th of an inch apart. We take the 50 scale and move the index over 5 graduations at a time. If the parallels are required to be $\frac{1}{12}$'th of an inch apart, we take the 60 scale and move the index over 5 graduations at a time. This method depends on the fact that by moving the index through 50 *graduations of the* 50 *scale*, 60 *graduations of the* 60 *scale*, and so on, we rule 2 lines which are parallel and *one inch apart*.

4. When every line in a drawing is some *constant* fraction of what it represents, the drawing is said to be "to scale," and the constant fraction is called the **Representative Fraction.**

Thus, if every line in the drawing is an 84th part of the corresponding line in the object represented, we say that

the scale is 84 inches to the inch,

i.e., 7ft. to the inch;

or that the Representative Fraction is $\frac{1}{84}$.

The Representative Fraction is therefore

$$\frac{\text{Any line in the drawing in inches}}{\text{What that line represents in inches}}.$$

5. Two other examples of diagonal scales are given, which the student should examine carefully.

(i.) *Make a diagonal scale to read miles and furlongs on a scale of* 20 *miles to the inch. Show* 100 *miles. Take off* 37*m.* 5*f. State the Representative Fraction.*

In this case the A, B, C of section 2 are

groups of 10 miles, 1 mile, 1 furlong.

$$\text{Now, as } \overset{\text{m.}}{20} : \overset{\text{m.}}{100} : 1'' : x''$$

$$\therefore \quad 20\,x = 100 \quad \therefore \quad x = 5''$$

We therefore make 8 horizontal spaces by drawing parallel lines 5 inches long. We draw the verticals half an inch apart (to show 10 miles), and subdivide the first division into 10 equal parts (Fig. iii.).

AB in the figure obviously represents 37 m. 5 f.

$$\left.\begin{array}{c}\text{The Representative}\\ \text{Fraction}\end{array}\right\} = \frac{1}{20 \times 1760 \times 36} = \frac{1}{1267200}$$

Notice.—In this scale, 20 miles being represented by 1 inch, 1 mile is represented by $\frac{1}{20}''$. The subdivision of that $\frac{1}{20}''$ into 8 equal parts, for furlongs, would be difficult in consequence of the parts being so extremely minute. But the diagonal scale avoids the difficulty without sacrificing anything.

(ii.) *Make a diagonal scale to read miles, furlongs, and chains, if* 3 *miles are shown by* $3\frac{1}{4}''$. *Assume* 7 *miles. Take off* 3 *m.* 7 *f.* 6 *ch., and state the Representative Fraction.*

Since 3 miles are shown by $3\frac{1}{4}''$, 1 mile is shown by $1\frac{1}{12}''$. Ten chains make a furlong. Therefore we must make 10

horizontal spaces, the verticals must be $1\frac{1}{12}''$ apart, and the first division must be subdivided into 8 equal parts.

AB in the figure (iv.) represents 3 m. 7 f. 6 ch.

$$\text{The Representative Fraction} = \frac{3\frac{1}{4}}{3 \times 1760 \times 36}$$

$$= \frac{13}{12 \times 1760 \times 36}$$

$$= \frac{13}{760320}$$

Note.—The verticals may be drawn $1\frac{1}{12}''$ apart by using the 60 scale and moving the index of the triangle over 65 *graduations.*

PLAIN SCALES.

6. The following examples will sufficiently indicate the method of constructing Plain Scales.

(i.) *The Representative Fraction being $\frac{1}{324}$, construct a plain scale to read yards. Show 70 yards.*

Since the Representative Fraction $= \frac{1}{324}$

$$\begin{aligned} \text{The scale is } 324'' &\text{ to the inch} \\ &= 27 \text{ ft. to the inch.} \\ &= 9 \text{ yds. to the inch.} \end{aligned}$$

$$\text{Now, as} \quad \overset{\text{yds.}}{9} : \overset{\text{yds.}}{70} :: 1'' : x''$$

$$\therefore \ 9x = 70''$$

$$\therefore \ x = 7\tfrac{7}{9}''.$$

We therefore draw a line $7\frac{7}{9}''$ long, divide it into 7 equal parts, so that each may show 10 yards (each part is $1\frac{1}{9}''$ long), and then subdivide the first division into 10 equal parts (each part is $\frac{1}{9}''$ long), to show single yards. (See Fig. v.)

N.B.—Above the diagonal scale on the Protractor are scales of 60, 50, 45, 40, 35, and 30. The primary divisions of these scales are thus $\frac{1}{6}''$, $\frac{1}{5}''$ $\frac{2}{9}''$, $\frac{1}{4}''$, $\frac{2}{7}$, and $\frac{1}{3}''$ respectively. We may therefore obtain *sevenths and ninths of an inch directly from the Instrument.*

(ii.) *The distance between two places is* 3 *miles, and is represented by* 1.5″. *Construct a plain scale to read miles and furlongs. Show* 10 *miles, and state the Representative Fraction.*

We must first find what length will represent 10 miles.

$$\text{As}\quad \overset{\text{m.}}{3} : \overset{\text{m.}}{10} :: 1\tfrac{1}{2}'' : x''$$

$$\therefore\quad 3x = \frac{10 \times 3}{2}$$

$$\therefore\quad x = 5''.$$

We now draw a line 5 inches long, divide it into 10 equal parts (each is ½″ long) for miles, and subdivide the first division into 8 equal parts for furlongs. (Fig. vi.)

$$\left.\begin{array}{c}\text{The Representative}\\ \text{Fraction}\end{array}\right\} = \frac{3}{2 \times 3 \times 1760 \times 36} = \frac{1}{126720}$$

(iii.) *The Representative Fraction is* $\frac{1}{63360}$. *Construct a Plain Scale to read miles and furlongs.*

Since the Representative Fraction $= \frac{1}{63360}$

$\therefore$ the scale is 63360″ to the inch.
= 5280 ft. to the inch.
= 1760 yds. to the inch.
= 1 mile to the inch.

As we are not told here to show any particular number of miles we can show as many as we choose.

Suppose that we show 6 miles.

We must draw a line 6 inches long. Each inch will

represent one mile, and we must subdivide the first division into 8 equal parts for furlongs.

See Fig. vii.

(iv.) *Construct a plain scale of* 25 *yards to the inch. Show* 150 *yards.*

We must first calculate what length of line will show 150 yards.

$$\text{As} \overset{\text{yds.}}{25} : \overset{\text{yds.}}{150} : 1'' : x''$$

$$\therefore \quad 25x = 150 \quad \therefore \quad x = 6''.$$

We now draw a line 6″ long. Subdivide into 15 equal parts (each will be $\frac{2}{5}''$), so that each part will show 10 yards; and subdivide the first division into 10 equal parts for single yards.

See Fig. viii.

Note.—The subdivision may be effected by the method of Problem 9 ("Practical Geometry").

COMPARATIVE SCALES.

7. Suppose that we have a foreign map or plan with the scale attached for some foreign measure—say kilometres or versts. A scale constructed *from the given scale* to read in some English or other different measure is called a *comparative scale.*

The following examples will sufficiently explain the method adopted.

(i.) *A French plan has a scale of decimetres,* 10 *to the inch, and a decimetre* = .327 *English feet. Make a comparative scale to read feet. Show* 20 *feet.*

The scale is 10 decimetres to the inch

= 3.27 feet to the inch,

and we are to show 20 feet.

$$\therefore \text{As } \overset{\text{ft.}}{3.27} : \overset{\text{ft.}}{20} :: 1'' : x''$$

$$\therefore 3.27\,x = 20 \qquad \therefore x = 6.11'' \text{ nearly.}$$

We therefore (Fig. ix.) draw a straight line 6.11″ long: bisect it (each part will show 10 feet), and subdivide the first division into 10 equal parts for single feet.

(ii.) *If* 60 *Russian versts are represented by* 7.5″, *and a verst* = 1167 *yds., draw a comparative scale to read miles. Show* 40 *miles.*

The scale is 60 versts to $7\frac{1}{2}''$

$$= 60 \times 1167 \text{ yds. to } 7\frac{1}{2}''$$

$$= \frac{60 \times 1167}{1760} \text{ miles to } 7\frac{1}{2}''.$$

Now, to find the length which represents 40 miles, we have

$$\text{As } \frac{60 \times 1167}{1760} : 40 :: \frac{15''}{2} : x''$$

$$\therefore \frac{60 \times 1167}{1760} x'' = 20 \times 15$$

$$x = \frac{20 \times 15 \times 1760}{60 \times 1167}$$

$$= \frac{8800}{1167} = 7.54'' \text{ nearly.}$$

We therefore (Fig. x.) draw a line 7·54″ long; divide it into 4 equal parts (each of these denotes 10 miles), and subdivide the first division into 10 equal parts for single miles.

(iii.) *Draw scales of* $\frac{3}{500}$ *to represent English feet, French metres, and Greek cubits, if* 1 *metre* = 3.27 *feet, and* 1 *cubit* = .45 *metres.*

The scale is 500″ to 3″ = 1″ to $\frac{3''}{500}$ = 1 ft. to $\frac{36''}{500}$ = 100 ft. to $\frac{36''}{5}$. We therefore (Fig. xi.) draw a line $7\frac{1}{5}''$ long (this will denote 100 ft.), and subdivide it in the usual manner.

Again, since 1 ft. is denoted by $\frac{36''}{500}$ $\therefore$ 3.27 ft. are denoted by $\frac{36 \times 3.27}{500}$ inches $= \frac{1.1772''}{5}$

But 3.27 ft. = 1 metre.

$\therefore$ 1 metre is denoted by $\frac{1.1772''}{5}$

$\therefore$ 10 metres are denoted by 2.3544″.

Now assume 40 metres. This will be denoted by 9.42″ nearly. Divide this into 4 equal parts, and subdivide the first division into 10 equal parts. See Fig. xii.

Again, since 1 cubit = .45 metres $\therefore$ 10 cubits = $4\frac{1}{2}$ metres, and are therefore denoted by $\frac{9}{2} \times \frac{1.1772''}{5}$

$$= 9 \times .11772''$$
$$= 1.05948$$
$$= 1.06'' \text{ nearly.}$$

Assume 50 cubits.

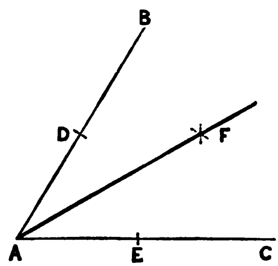
This will be denoted by 5.3″.

And the scale may be constructed (Fig. xiii.) in the usual manner.

PRACTICAL GEOMETRY.

Note.—Given lines must be drawn *light*, construction lines *dotted*, required lines *dark*.

1. *To bisect a given rectilineal angle.*



Let BAC be the given angle.

With center A and any convenient radius describe an arc cutting AB, AC in D and E respectively.

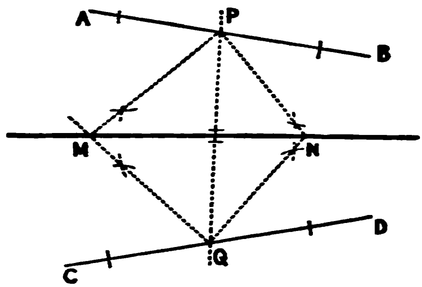
With D and E as centers and any convenient radius (the same for each), describe arcs intersecting in F.

Join AF.

This will bisect the angle BAC.

Proof.—Apply Euc. I. 8.

2. *To draw a straight line which would bisect the angle formed by two intersecting straight lines when their point of section is inaccessible.*



Let AB, CD be two converging straight lines, and suppose their intersection inaccessible.

Draw *any* straight line PQ cutting AB, CD in P and Q.

Bisect the angles at P and Q as in the figure, and let these bisectors meet in M and N respectively.

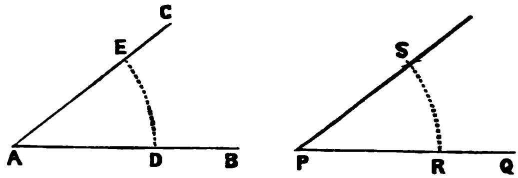
Join MN.

MN is the straight line required.

Note.—Suppose R the inaccessible point of section. Then N is the center of the circle *inscribed* in the Δ PRQ, and M is the center of the circle (*escribed*) which touches PQ, PA, QC.

By means of the Property that "Two Tangents drawn to a circle from the same external point are equal," and by applying Euc. I. 8, it follows that RM and RN *each* bisect the angle at R. Hence MN passes through R and bisects the angle at that point.

3. *At a given point in a given straight line to make an angle which shall be equal to a given angle.*



Let BAC be the given angle,
PQ the given straight line,
and P the given point in it.

It is required to make at the point P an angle equal to BAC.

With center A and any radius (not too small) describe an arc cutting AB, AC, in D and E respectively.

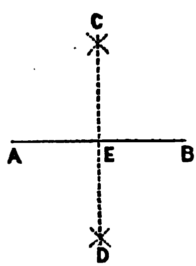
With center P and the same radius describe an arc cutting PQ in R.

Measure off RS = DE and join PS.

RPS is the angle required.

Proof.—Apply Euc. III. 28, and III. 27.

4. *To bisect a given finite straight line.*



Let AB be the given straight line.

With center A and any convenient radius describe an arc, and with center B and the same radius describe another arc, and let the two arcs intersect in C, D.

Join CD, and let it cut AB in E.

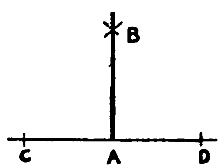
Then AB is bisected in E.

Proof.—Apply Euc. I. 8, and I. 4.

Note.—CD is also at right angles to AB.

5. *To draw a straight line at right angles to a given straight line from a given point in the same.*

Method i.—(To be used when the given point is not very near the end of the line.)



Let A be the given point in CD, the given straight line.

Make AC = AD.

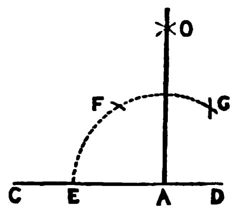
With centers C and D and the same radius draw arcs intersecting in B.

Join AB.

AB is at right angles to CD.

Proof.—Euc. I. 8.

Method ii. (When the point is very near the end of the line, and we are not allowed to produce the line.)



With center A and any radius describe the arc EFG. Let this cut CD in E, and make EF and FG each equal to the radius.

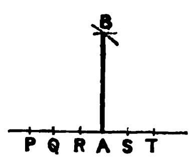
With centers F and G and any the same radius describe arcs intersecting in O.

OA is at right angles to CD.

Proof.—FAE = DAG, for either = 60° (Euc. I. 1),
and FAO = GAO (Euc. I. 8),
∴ EAO = DAO, (by addition.)
Whence AO is at right angles to CD.

Note.—Euc. I. 1 shows that a chord equal to the radius subtends an angle of 60° at the center of the circle.

Method iii. (When the point is not very near the end of the line.)



Take AR = RQ = QP = AS = ST.
With center P and radius = PT describe an arc.

With center A and radius = PS describe another arc.

Let the two arcs intersect in B.

BA is at right angles to PT.

Proof.—Let each of the equal distances PQ, QR $= x$

Then PB $= 5x$ PA $= 3x$ and AB $= 4x$

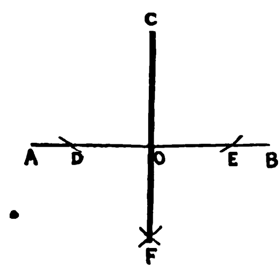
$$\therefore PA^2 + AB^2 = 9x^2 + 16x^2 = 25x^2$$
$$= (5x)^2$$
$$= PB^2$$

$\therefore$ BAP is a right angle (Euc. I. 48.)

Note.—Method i. is, of course, simpler than Method iii.; but the latter is useful as illustrating the fact that *when the sides of a triangle are in the proportion* 3, 4, *and* 5 *the angle subtended by the longer side is a right angle.*

6. *To draw a straight line perpendicular to a given straight line from a given external point.*

Method i. (When the point is not nearly above the end of the line.)



Let AB be the given straight line, C the given external point.

With center C and any convenient radius describe an arc cutting AB in D and E.

With centers D and E and any (the same) convenient radius describe arcs intersecting in F.

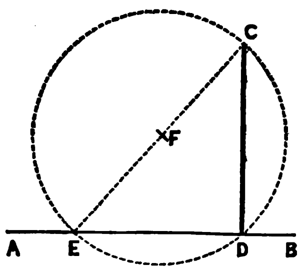
Join C, F.

CF is the perpendicular required.

Proof.—Apply Euc. I. 8 to the $\triangle$s CDF, CEF, so obtaining DCF = ECF.

Whence by Euc. I. 4. DOC = EOC.

Method ii. (When the external point is nearly above the end of the line.)



Let C be the given external point as before.

Take E any point in AB.

Join CE and bisect it in F.

With center F and radius FE describe a circle.

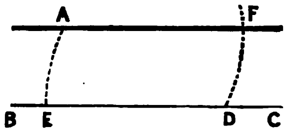
Let this cut AB again in D.

Join CD.

CD is the perpendicular required.

Proof.—EDC is the angle in a semicircle, and is therefore a right angle. (Euc. III. 31.)

7. *Through a given point to draw a straight line parallel to a given straight line.*



Let A be the given point, and BC the given straight line.

In BC take any point D.

With D as center and radius DA describe the arc AE, cutting BC in E.

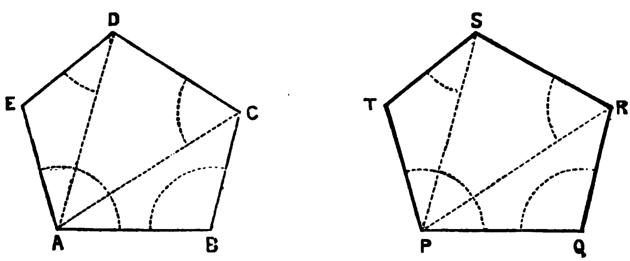
With center A and radius AD describe an arc DF, and make DF = AE.

Join AF.

AF will be parallel to BC.

Proof.—It is easily seen that the alternate angles EDA, DAF are equal.

8. *On a given straight line to make a figure similar (i.e. equiangular) to a given rectilineal figure.*



Let ABCDE be the given figure,

and PQ the given straight line.

Join AC, AD, and make the angles at P, equal respectively to the angles at A.

Make the angle at Q = the angle at B;

Thus the point R is fixed.

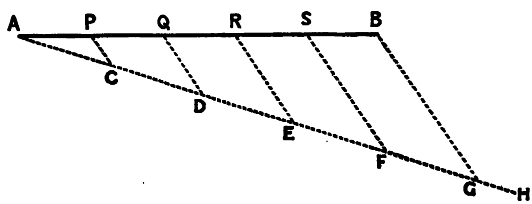
Make PRS = ACD;

Thus the point S is fixed.

Make PST = ADE;

Thus the point T is fixed.

9. *To divide a straight line into any number of equal parts (say five).*



Let AB be the given straight line.

Through A draw any other straight line, AH.

Mark off on this five equal distances, AC, CD, DE, EF, FG.

Join BG.

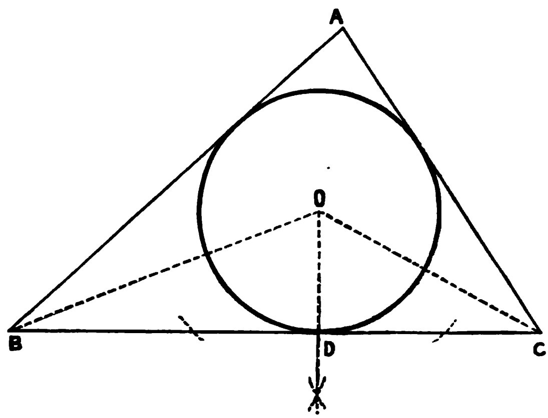
Draw FS, ER, DQ, CP parallel to BG.

AB is divided as required.

Proof.—Apply Euc. VI. 2.

Note.—If the line is to be divided into 2^2, 2^3, 2^4, or generally 2^n equal parts, it may be done by *successive bisection.*

10. *To inscribe a circle in a given triangle.*



Let ABC be the given triangle.

Bisect the angles at B and C.

Let these bisectors intersect in O.

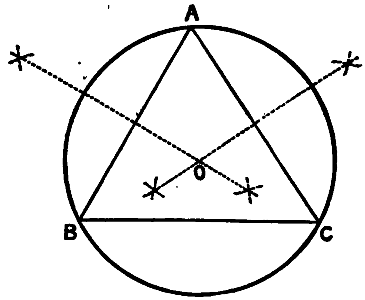
Draw OD perpendicular to BC.

With center O and radius OD describe a circle.

This will be the circle required.

Proof.—Euc. IV. 4 is identical with this problem.

11. *To describe a circle about a given triangle.*



Let ABC be the given triangle.

Bisect 2 sides AB, AC.

Let the 2 bisectors meet in O.

With center O and radius OA describe a circle.

This will be the circle required.

Note.—This problem is the 5th Proposition of Euclid's Fourth Book.

12. By Euc. I. 32, Cor. 1, it is proved that

"All the Interior angles of any Rectilineal figure, "together with four right angles, are equal to "twice as many right angles as the figure has "sides."

If, therefore, we suppose the polygon to have n sides,

$$\text{All its interior angles} + 4.90 = 2n.90$$

$$\therefore \text{All the interior angles} = 2n.90 - 4.90$$

$$= (2n - 4)\ 90$$

$$= (n - 2)\ 180.$$

Again, if the polygon be **Regular**—that is, if it be equilateral *and consequently equiangular*—each of the n angles will be

$$\frac{(n-2)\,180^\circ}{n}.$$

Thus each interior angle of a regular *pentagon* will be

$$\frac{(5-2)\,180^\circ}{5} = 108^\circ,$$

and each interior angle of a regular hexagon will be

$$\frac{(6-2)\,180^\circ}{6} = 120^\circ.$$

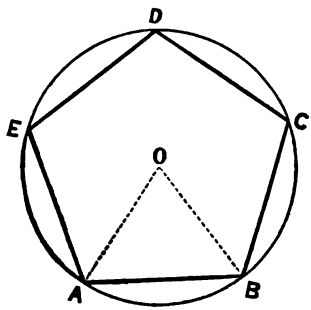
13. Moreover, if the polygon be regular, a circle can be described about it, and the angles subtended by each side of the polygon at the center of the circumscribing circle will be all equal, each of them being $\frac{360}{n}$.

For, by Euc. I. 15, Cor. ii., the *sum of all* the angles at the center $= 360^\circ$, and since they are all equal, *each* of them $= \frac{360^\circ}{n}$.

Such an angle may be called the *central angle* of the polygon.

14. *To inscribe a regular polygon of n sides in a given circle.*

Take $n = 5$.



Let ABC be the given circle and O its center.

Draw any radius OA.

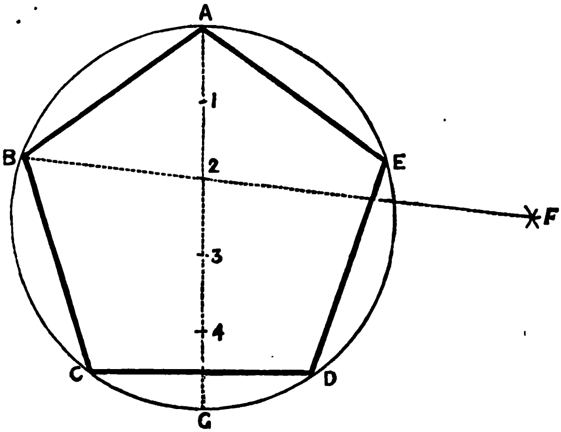
Make the angle AOB = the central angle of the pentagon, viz., $\frac{360°}{5} = 72°$.

Join AB.

Step off distances BC, CD, DE, each equal to AB.

ABCDE will be the pentagon required.

15. *To inscribe a regular polygon of any number of sides (say five) in a given circle* **without using the Protractor.**



Draw any diameter AG.

Divide this into five equal parts.

With centers A and G, and with radii each equal to AG, describe arcs intersecting in F.

Join F with the *second* point of division, measuring from A, on the diameter AG.

Produce this to meet the circle in B.

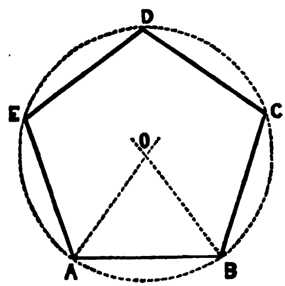
Join AB, and step off distances BC, CD, DE, each equal to AB.

ABCDE is the regular pentagon required.

Note.—(i.) F must be joined with the *second point of division, whatever be the number of the sides of the polygon.*

(ii.) The method of this Problem is not capable of *rigid mathematical proof*. It may, however, be regarded as fairly accurate in *practice*.

16. *To describe a regular polygon of n sides on a given straight line.*



Take $n = 5$, and let AB be the given straight line.

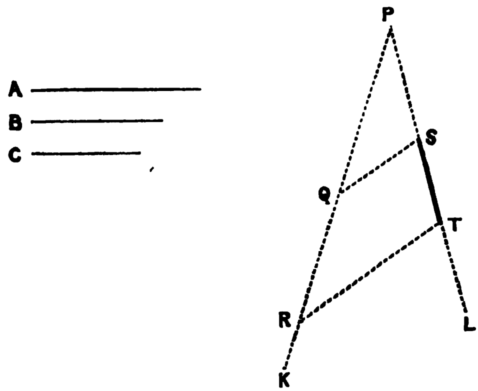
Make BAO, ABO each equal to the semi-interior angle of a regular pentagon—viz. $\frac{1}{2} \cdot \frac{(5-2)\,180^\circ}{5} = 54^\circ$.

With center O and radius OA or OB describe a circle.

Step off BC, CD, DE, each equal to AB.

ABCDE will be the pentagon required.

17. *To find a fourth proportional to three given straight lines.*



Let A, B, C, *in order*, be the three given straight lines.

Draw two straight lines PK, PL making any angle.

Make PQ = A, QR = B, and PS = C.

Join QS and draw RT parallel to QS.

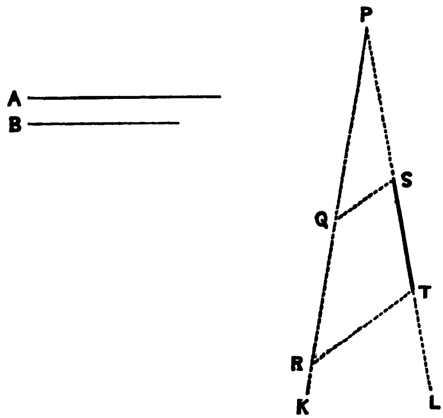
ST is the fourth proportional required.

i.e. A : B :: C : ST.

Cor.—The Rect. A. ST = the Rect. B. C.

Proof.—Euc. VI. 2.

18. *To find a third proportional to two given straight lines.*



Let A and B be the two given straight lines.

Draw PK, PL making any angle.

Make PQ = A, QR = B, and PS = B.

Join QS and draw RT parallel to it.

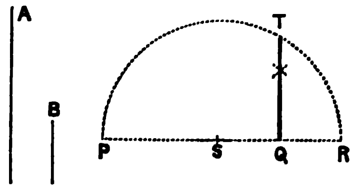
ST is the third proportional required.

(*i.e.*) A : B :: B : ST.

Cor.—The Rect. A . ST = B^2.

Proof.—Euc. VI. 2.

19. *To find a mean proportional between two given straight lines.*



Let A and B be the two given straight lines.

Make PQ = A, and QR = B.

On PR describe a semicircle, and draw QT at right angles to PR, meeting the circumference in T.

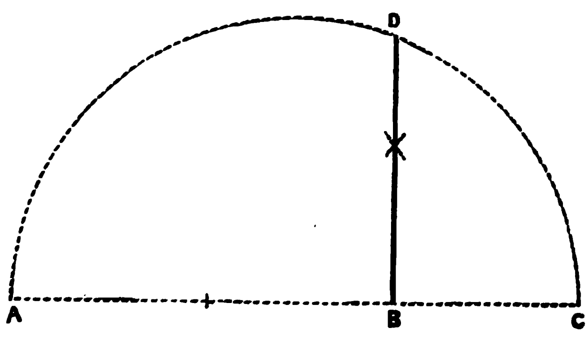
QT is the mean proportional required.

(*i.e.*) A : QT : QT : B.

Cor.—The Rect. A . B = QT^2.

Proof.—Euc. VI. 8.

20. *To make a straight line to represent $\sqrt{2}$ inches, or $\sqrt{\frac{2}{3}}$ inches, or similar quantities.*



(i.) To make a straight line $\sqrt{2}$ inches long.

Make AB = 2″, and BC = 1″.

Take BD a mean proportional between them.

Then $BD^2 = AB.\ BC = 2.1 = 2$.

$\therefore\ BD = \sqrt{2}''$.

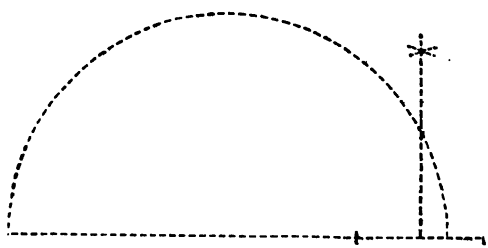
(ii.) To make a straight line = $\sqrt{\frac{2}{3}}$ inches long.

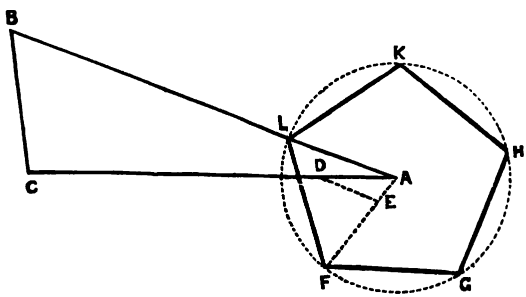
Make AB = $\frac{2}{3}''$, and BC = 1″,

and take BD, a mean proportional between them.

BD will be $\sqrt{\frac{2}{3}}$ inches long.

21. *To make a regular polygon of n sides (say, $n = 5$) equal to a given triangle.*





Let ABC be the given triangle.

Make BAF $= \frac{360^\circ}{5} = 72^\circ$.

Take AD, one fifth of AC, and draw DE parallel to AB.
Let this meet AF in E.

Take AL, a mean proportional between AB and AE.
With center A and radius AL describe a circle.
Let this cut AF in F.

Join LF, and step off chords FG, GH, HK, KL, each equal to LF,

and the polygon required is constructed.

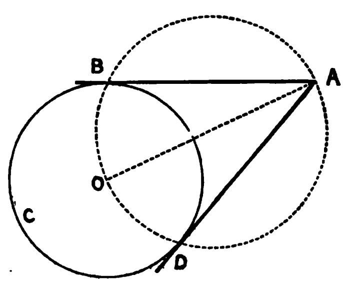
Proof.—Since BAE is common to the two triangles BAE, LAF,

and BA : AF : LA : AE (const.)

$\therefore$ the triangle LAF = the triangle BAE (Euc. VI. 15).
= the triangle BDA (Euc. I. 37).
= one fifth of the triangle ABC,

whence the proof is obvious.

22. *To draw a tangent to a circle from an external point.*



Let A be the given external point,
and O the center of the given circle.
Join AO, and on it as diameter describe a circle,
cutting the given circle in B and D.
Join AB.
This is the tangent required.

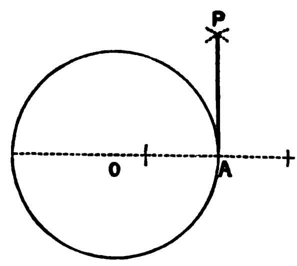
Cor.—If AD be joined, this is another tangent to the circle, and AB = AD.

Note.—(i.) A tangent to a circle is a straight line which meets the circle but does not cut it when produced.

(ii.) A straight line will meet the circle and not cut it—*i.e.*, will be a tangent, if it be at right angles to a diameter at its extremity.

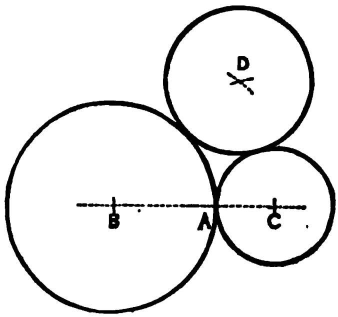
(iii.) In the figure above, AB is at right angles to OB—since OBA is the angle in a semi-circle.

23. *To draw a tangent to a circle from a point on its circumference.*



Let A be the point on the circumference.
O the center of the circle.
Join OA and draw AP at right angles to it.
AP is the tangent required.

24. *To draw three circles of given radii, say $\frac{5}{9}''$, $\frac{2}{7}''$, and $\frac{2}{5}''$, to touch each other.*



Take any straight line and A any point in it.
Make AB $= \frac{5}{9}''$ and AC $= \frac{2}{7}''$.

With center B and radius BA describe a circle, and with center C and radius CA describe another circle.

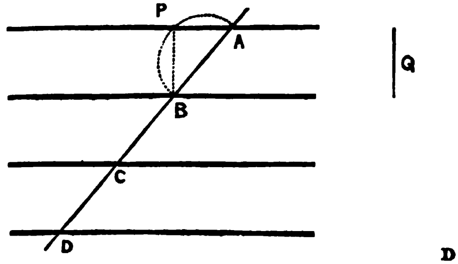
With center B and radius $= \frac{5}{9}'' + \frac{2}{5}''$, describe an arc, and with center C and radius $= \frac{2}{7}'' + \frac{2}{5}''$ describe another arc.

Let these two arcs intersect in D.

With center D and radius $= \frac{2}{5}''$ describe a circle.

This will touch each of the other circles

25. *To draw a series of parallel straight lines a given distance apart, through points ranged equidistantly along a straight line.*



Let A, B, C, D be the points ranged equidistantly along the given straight line, and Q the given distance.

Describe a semi-circle on AB.

In it place BP = Q. (Euc. IV. 1.)

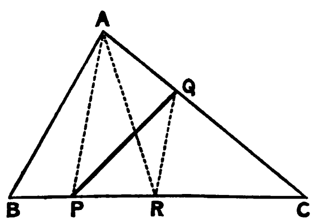
Join AP, and produce it both ways.

Through B, C, D draw straight lines parallel to AP.

These will be the parallels required.

Note.—Q evidently must not be greater than AB, for we cannot place in the semi-circle a straight line greater than its diameter. Hence by drawing straight lines through A, B, C, D, at right angles to the line in which they are, we shall obtain *that series of parallels through the given points which are at the maximum distance apart.*

26. *To bisect a triangle by a straight line drawn from a given point in one of the sides.*



Let ABC be the given triangle,

P the given point in the side BC.

Take R, the middle point of BC.

Join AP, AR.

Draw RQ parallel to AP.

Join PQ.

This will bisect the triangle.

Proof.—The triangle AQP = the triangle ARP (Euc. I. 37).

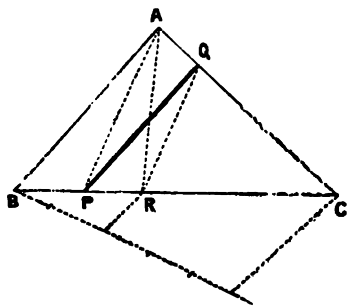
To each add the triangle ABP.

$\therefore$ the Fig. ABPQ = the triangle ABR

= $\frac{1}{2}$ (the triangle ABC)

for BR = RC.

27. *To divide a triangle into two parts which shall be in a given ratio, (say 2 : 3) by a straight line drawn from a given point in one of the sides.*



Let ABC be the given triangle,

P the given point in BC.

Take R in BC, such BR : RC :: 2 3.

Join AP, AR.

Draw RQ parallel to AP.

Join PQ.

This will divide the triangle as required.

Proof.—As in the preceding problem, we may prove that

Fig. ABPQ = the triangle ABR.

But the triangle ABR : triangle ABC :: BR : BC

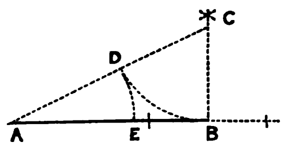
:: 2 : 5

$\therefore$ Fig. ABPQ = $\frac{2}{5}$ of the triangle ABC.

Hence the triangle QPC = $\frac{3}{5}$ of the triangle ABC.

$\therefore$ Fig. ABPQ : the triangle QPC :: 2 : 3.

28. *To divide a straight line into extreme and mean ratio, i.e., into two parts, such that the greater part is a mean proportional between the whole and the lesser part.*



Let AB be the given straight line.

Draw BC at right angles to AB.

Make BC = $\frac{1}{2}$ AB.

Join AC.

With center C and radius CB describe a circle.

Let this cut AC in D.

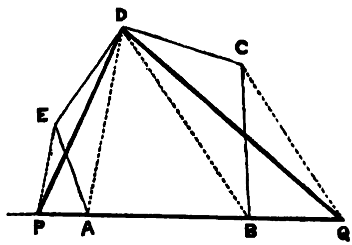
With center A and radius AD describe a circle.

Let this cut AB in E.

AB is divided as required in the point E.

Proof.—This proposition is Euc. II. 11, and the figure is merely a modification of that in the text.

29. *To reduce a polygon into an equal triangle.*



Let ABCDE be the given polygon.

Join A, one extremity of the base AB, with the next angular point but one (D).

Through E (the omitted point) draw EP parallel to AD.

Join DP.

Then since the triangle DPA = the triangle DEA,

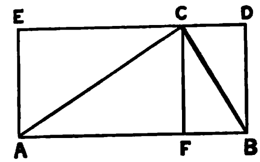
Add to each the figure DABC.

∴ the *four*-sided figure DPBC = the *five*-sided figure ABCDE.

Hence the process adopted reduces the original figure into another of equal area which has the number of its sides *one less.*

By repeating the process, therefore, as often as necessary, it is clear that we shall finally obtain a TRIANGLE (*in this case* DPQ) equal in area to the original polygon.

30. *The area of any triangle* $=$ ($\frac{1}{2}$ *base*) $\times$ *altitude.*



Let ABC be any triangle.

Construct the rectangle ABDE as in the figure.

Draw CF perpendicular to AB.

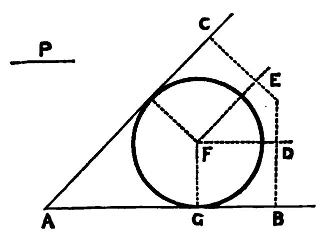
Then the triangle ABC $= \frac{1}{2}$ the rectangle EB. (Euc. I. 41.)

$= \frac{1}{2}$ AB. AE

$= (\frac{1}{2}$ AB) (CF)

$= (\frac{1}{2}$ base) $\times$ altitude.

31. *To draw a circle of given radius to touch two given intersecting straight lines.*



Let AB, AC be the given intersecting straight lines.

P the given radius.

In AB take any point B.

Draw BD at right angles to AB, making BD = P.

Draw DF parallel to AB.

In AC take any point C.

Draw CE at right angles to AC making CE = P.

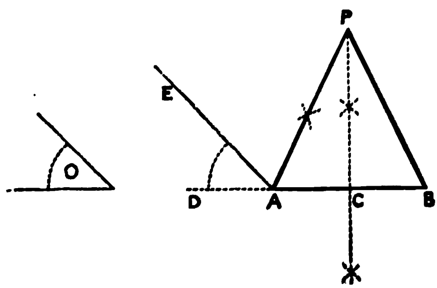
Draw EF parallel to AC.

Let this cut DF in F.

With center F and radius = P describe a circle.

This will be the circle required.

32. *On a given base to make an isosceles triangle having a given vertical angle.*



Let AB be the given base, and O the given angle.

Produce BA, and make the angle DAE = the angle O.

Bisect the angle EAB by AP.

Bisect AB at right angles by CP.

Join BP.

APB is the isosceles trangle required.

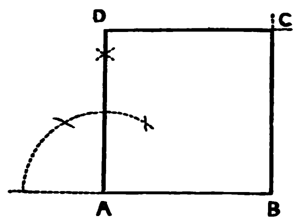
Proof.—By Euc. I. 4 BP = AP ∴ the triangle is isosceles and the angles PAB, PBA, APB together = PAB, PAE, EAD together, for the sum of either three = two right angles.

But the first two on the one side = the first two on the other side.

∴ APB = EAD.

= the angle O.

33. *To make a square on a given straight line.*



Let AB be the given straight line.

Draw AD at right angles to AB.

Make AD = AB.

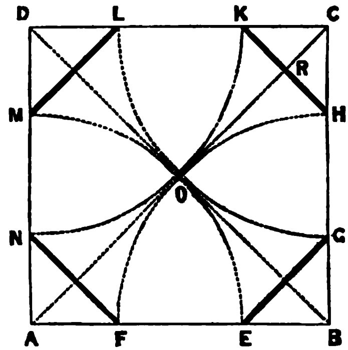
With centers D and B and radii equal to AB, describe arcs.

Let these intersect in C.

Join BC, DC.

ABCD is obviously the square required.

34. *To inscribe an octagon in a given square.*



Let ABCD be the given square.

Draw the two diagonals intersecting in O.

With centers A, B, C, D, and radius equal to OA, describe arcs cutting the sides of the square in E, F, G, H, K, L, M, N,

EFGHKLMN is the octagon required.

Proof.—Draw OY perpendicular to DC.

Then angle KOC = angle KNO in the alternate segment.

= $\frac{1}{2}$ angle CDO (at the center).

= $\frac{1}{2}$ angle YOC.

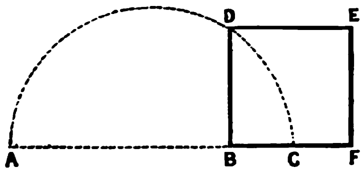
= angle KOY.

∴ KY = KR.

∴ LK = KH.

Hence the octagon is equilateral.

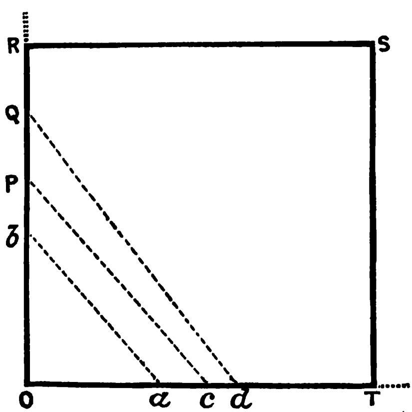
35. *To reduce a triangle into an equal square.*



Take BC = half-base and AB the altitude of the triangle.
Make BD a mean proportional between AB and BC.
On BD describe a square.
This will be the square required.

Proof.—For $BD^2 = BC, AB$
$= (\frac{1}{2} \text{ base}) \times \text{altitude}$
= area of given triangle.

36. *To make a square equal to the sum of any number of given squares.*



Let A, B, C, D be the *sides* of the given squares arranged in ascending order of magnitude—*i.e.*, let A be less than B, B than C, and C than D.

Draw two straight lines at right angles to each other.

Let them intersect in O.

Make $Oa = A$; $Ob = B$ and join a, b.

Make $OP = ab$; and $Oc = C$ and join Pc.

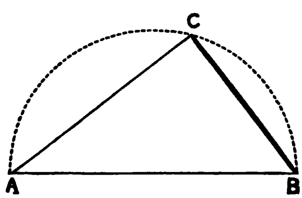
Make $OQ = Pc$ and $Od = D$ and join Qd.

Make OR, OT each $= Qd$ and complete the square.

ORST is the square required.

Proof.—$OR^2 = Qd^2 = QO^2 + Od^2 = Pc^2 + D^2 = OP^2 + OC^2 + D^2 = ab^2 + C^2 + D^2 = Ob^2 + Oa^2 + C^2 + D^2 = A^2 + B^2 + C^2 + D^2$.

37. *To make a square equal to the difference of two given squares.*



Let AB be the side of the larger of the two given squares.

On it describe a semicircle.

In the semicircle place a straight line AC, equal to the side of the other given square.

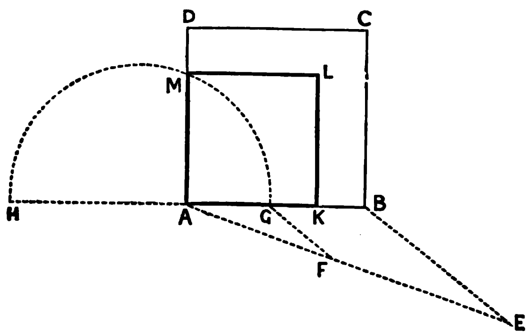
Join BC.

The square on BC will be the square required.

Proof.—Apply Euc. I. 47.

38. *To make a square which shall be any given part or multiple of a given square.*

(*E.g.* Make a square which shall be $\frac{3}{7}$'ths of the given square.)



Let AB be the side of the given square.

Make AG the required part of AB ($\frac{3}{7}$'ths).

Produce BA, and make AH = AB.

On HG describe a semicircle cutting AD in M.

On AM construct the square AKLM.

This is the square required.

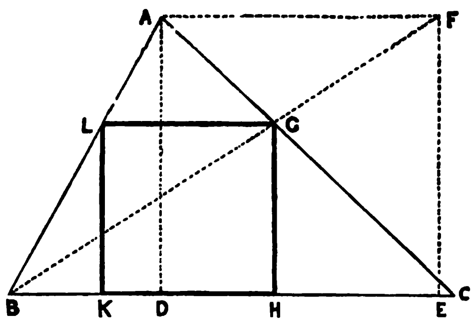
Proof.—AM being a mean proportional between AG and AB, we have

AB : AG :: fig. on AB : sim. fig. on AK

(Euc. VI. 20, Cor. 2).

Hence square on AK : square on AB :: AG : AB :: 3 : 7.

39. *To inscribe a square in a triangle.*



Let ABC be the given triangle.

Draw AD perpendicular to BC.

On AD describe the square ADEF.

Join BF, and let it cut AC in G.

Draw GH parallel to AD, GL parallel to BC,

and LK parallel to AD.

GHKL is the square required.

Proof.—By similar triangles LG : AF :: BL : BA

:: KL : AD

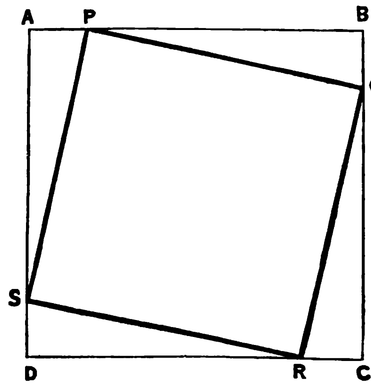
But AF = AD.

∴ LG = KL,

and GHKL is a parallelogram by construction.

∴ all its four sides are equal, &c.

40. *To construct a square within a square, having one of its angular points at a given point in the side thereof.*



Let ABCD be the given square.

P the given point in one of the sides AB.

Mark off BQ, CR, DS, as in the figure, each equal to AP.

PQRS is the square required.

Proof.—By applying Euc. i. 4 to the four triangles, it is seen that the figure PQRS is *equilateral*, and that *the triangles are equiangular.*

Again, the angles ASP, APS are together equal to a right angle, and APS = DSR.

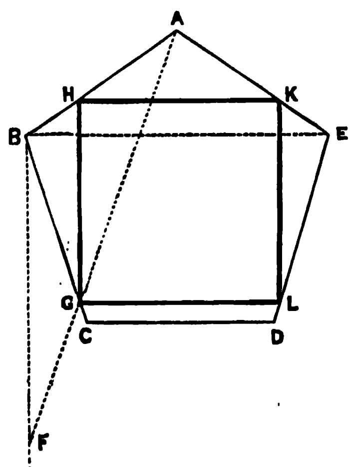
∴ the angles ASP, DSR are together equal to a right angle.

Whence the angle PSR is a right angle.

Similarly for the other angles of the figure PQRS.

∴ it is a square.

41. *To inscribe a square in a regular pentagon.*



Let ABCDE be the given regular pentagon.

Join any angular point B with the next angular point but one (E).

Draw BF at right angles to BE, and make BF = BE.

Join AF, cutting BC in G.

Draw GH parallel to BF, HK parallel to BE, KL parallel to BF, and join GL.

GHKL is the square required.

Proof.—As in Proposition 39, we may prove that GH = HK.

Now, since the pentagon is *regular* AB = AE, and HK being parallel to BE,

$$AH = AK,$$

$$\therefore BH = KE.$$

Hence, applying Euc. I. 26, to the triangles HBG, KEL, we have KL = HG.

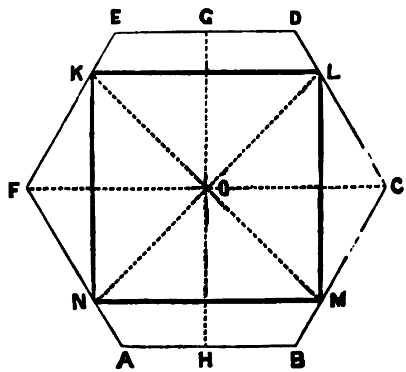
Also KL is parallel to HG.

∴ GL is equal and parallel to HK.

∴ GHKL is a right-angled parallelogram, and HK = HG.

∴ the figure is a square. Q. E. D.

42. *To inscribe a square in a regular hexagon.*



Let ABCDEF be the regular hexagon.

Join FC, and bisect it at right angles in O.

Bisect each of the four right angles at O by the straight lines KM, LN, as in the figure.

KLMN is the square required.

Proof.—Apply Euc. I. 26 to the triangles KOF, LOC.

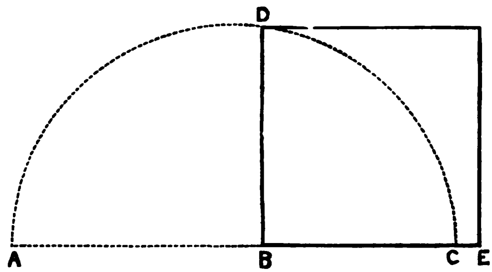
Hence KO = OL.

Similarly OK = ON.

Hence OK, OL, OM, ON are all equal.

Whence it is easily seen that the figure KLMN is both equilateral and rectangular.

43. *To construct a square having a given area, say 1.35″.*



Make AB = 1.35″, and BC = 1″.

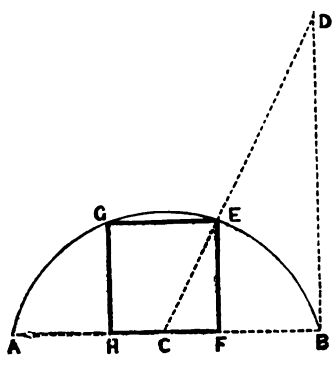
Take BD a mean proportional between them.

On BD describe a square.

This is the square required.

Proof.—$BD^2 = AB.BC = 1.35'' \times 1'' = 1.35$ square inches.

44. *To inscribe a square in a segment of a circle.*



Let AEB be the segment.
Bisect AB the base in C.
Draw BD at right angles to AB, making BD = AB.
Join DC, and let this cut the arc in E.
Draw EF parallel to BD, and EG parallel to AB.
Let the latter cut the arc in G.
Draw GH parallel to EF.
Then GEFH is the square required.

Proof.—CEF, CDB are similar triangles, and BD = 2 CB.

$\therefore$ FE = 2 CF.

By applying Euc. III. 3, we may prove that

GE = 2 CF.

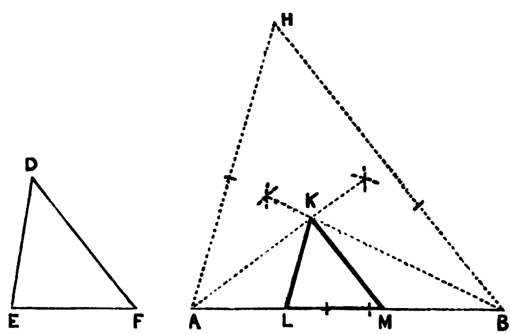
$\therefore$ FE = GE.

Also the figure GEFH is a right-angled parallelogram by construction, and has its opposite sides equal.

$\therefore$ it is a square,

And it is inscribed in the given segment.

45. *To make a triangle having a given perimeter and similar to a given triangle.*



Let AB be the given perimeter,

and DEF the given triangle.

Make the angles at A and B equal to the angles at E and F respectively.

Then the triangle AHB is similar to the triangle DEF.

Bisect the angles at A and B by straight lines meeting in K.

Draw KL, KM, parallel to HA, HB respectively.

KLM is the triangle required.

Proof.—The angle KLM = the angle HAB (Euc. I. 29) = angle at E.

So the angle KML = the angle at F.

∴ the triangle KLM is similar to the triangle DEF.

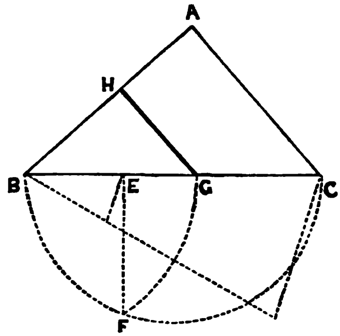
Again, the angle LKA = the angle HAK by alternate angles.

= the angle LAK (const.)

∴ LK = LA.

So MK = MB ∴ the triangle KLM has the given perimeter.

46. *From a given triangle to cut off any fractional part (say one-third).*



Let ABC be the given triangle.

On BC describe a semicircle.

Make BE = $\frac{1}{3}$ BC, and draw EF at right angles to BC.

Let this cut the semicircle in F.

With center B and radius BF describe an arc.

Let this cut BC in G.

Draw GH parallel to AC.

The triangle HBG shall be the part required.

Proof.—BC, BG, BE are proportionals.

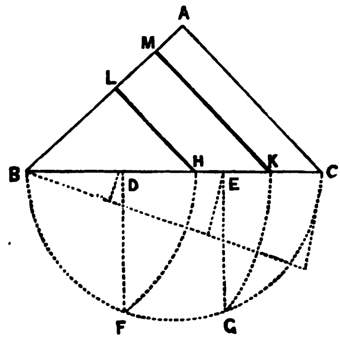
$\therefore$ as BC : BE :: triangle on BC : similar triangle on BG.

But BC = 3BE (Euc. VI. 19, Cor.)

$\therefore$ triangle ABC = 3 triangle HBG.

i.e., triangle HBG = $\frac{1}{3}$ triangle ABC.

47. ***To divide a triangle into any number of equal parts (say three) by straight lines drawn parallel to one of the sides.***



Let ABC be the triangle.

On BC describe a semicircle.

Divide BC into three equal parts in D and E.

Draw DF, EG, at right angles to BC.

Let them meet the semicircle in F and G.

With center B and radii BF, BG, describe arcs.

Let these cut BC in H and K.

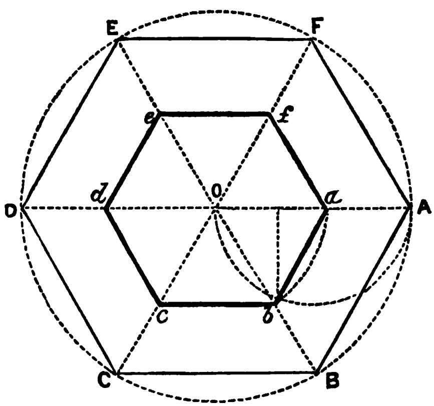
Draw HL, KM parallel to CA.

These divide the triangle as required.

Proof.—As in the last problem, HBL $= \frac{1}{3}$ of ABC, and similarly MBK $= \frac{2}{3}$ of ABC.

$\therefore$ LHKM $= \frac{1}{3}$ of ABC. $\therefore$ &c.

48. *To make a polygon similar to a given polygon, its area being any fractional part (say one-third) of that of the given polygon.*



Take the case of a regular hexagon ABCDEF in the figure above.

Cut off a triangle Oab $= \frac{1}{3}$ of the triangle OAB, ab being parallel to AB (Prob. 42).

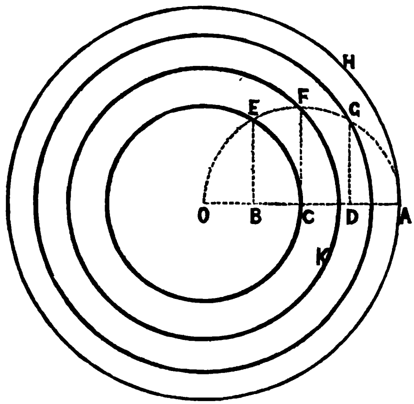
From b draw bc parallel to BC, meeting OC in c.

From c draw cd parallel to CD.

By this method complete the figure abcdef.

This is obviously the figure required.

49. *To divide a circle into any number of equal concentric annuli (say four).*



Draw any radius OA, and divide it into four equal parts at B, C and D.

On OA as diameter describe a semicircle.

Draw BE, CF, DG at right angles to OA.

Let them meet the semicircle in EFG.

With center O and radii OE, OF, OG describe circles.

These divide the given circle as required.

Proof.—

$$\frac{\text{The circle OA}}{\text{The circle EC}} = \frac{\pi\,OA^2}{\pi\,OE^2} = \frac{OA^2}{OE^2} = \frac{OA^2}{OA.\,OB} = \frac{OA}{OB} = \frac{4}{1}$$

$\therefore$ the circle EC $= \frac{1}{4}$ of the circle HA.

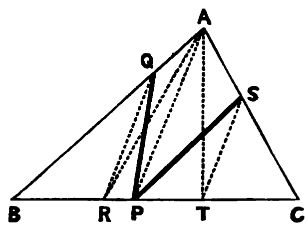
So the circle FK $= \frac{3}{4}$ of the circle HA.

$\therefore$ the annulus EFK $= \frac{1}{4}$ of the circle HA, and so on.

Note.—(i.) The area of any circle $= \pi \times$ (the radius)^2 where $\pi = 3.14159$ nearly, and $OE^2 = OA.\ OB.$ (Euc. VI. 8, Cor.)

(ii). The circle being the limiting form of a polygon when the number of sides is indefinitely increased, this problem is really a case of Prob. 48.

50. *To draw straight lines from a given point in one of the sides, dividing a given triangle into any number of equal parts, say three*).



Let P be the given point in BC, one of the sides of the triangle ABC.

Divide BC into three equal parts in RT.

Join AR, AP.

Through R draw RQ parallel to AP.

Let this meet AB in Q.

Join PQ.

Then QBP $= \frac{1}{3}$ of ABC (Prob. 27).

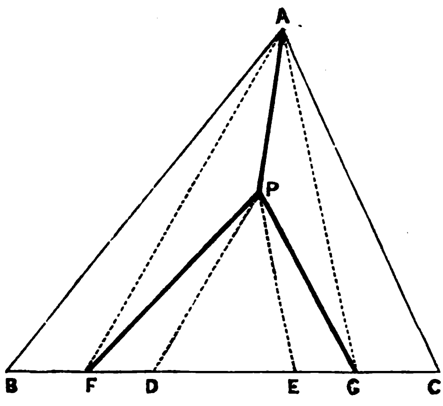
Again, Join A, T. Draw TS parallel to AP, meeting AC in S.

Join PS.

Then PCS $= \frac{1}{3}$ of ABC (as in Prob. 27).

$\therefore$ PQ, PS are the straight lines required.

51. *To draw straight lines from a given point within a triangle, so as to divide it into any number of equal parts (say three).*



Let P be the given point.

Divide BC into three equal parts in D, E.

Join PD, PE, and draw AF, AG, parallel to PD, PE respectively.

Join PA, PF, PG.

These divide the triangle into three equal parts as required.

Proof.—The triangle APF = the triangle ADF (Euc. I. 37).

Add to each the triangle ABF.

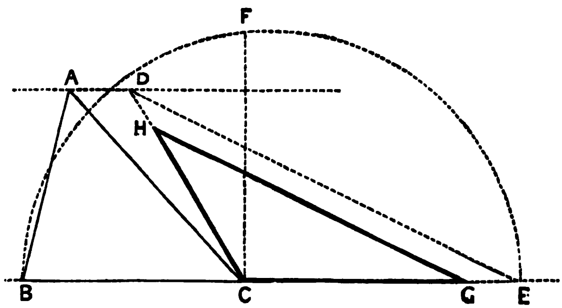
∴ the figure APFB = the triangle ADB.

= $\frac{1}{3}$ of the triangle ABC.

So the figure APGC = $\frac{1}{3}$ of the triangle ABC.

∴ the remaining triangle PFG = $\frac{1}{3}$ of the triangle ABC.

52. *To make a triangle equal to one triangle and similar to another.*



Let ABC be the triangle to which the required triangle must be equal.

Draw AD parallel to BC.

Make the angle ECD equal to one angle of the triangle to which the required triangle must be similar.

At D make the angle CDE equal to another angle of the same triangle.

We have therefore to make a triangle equal to ABC and similar to CDE.

Take CF a mean proportional between BC and CE.

Make CG = CF.

Draw GH parallel to ED.

CGH is the triangle required.

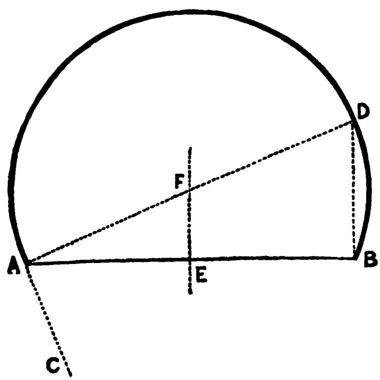
Proof.—CE, CG, BC are proportionals,

$\therefore$ CE : BC :: figure on CE : figure on CG.

$\therefore$ triangle CDE : triangle ABC :: triangle DCE : triangle CGH.

$\therefore$ &c.

53. *On a given straight line to make a segment of a circle containing an angle equal to a given angle.*



Let AB be the given straight line.

Make the angle BAC = the given angle.

Draw AD at right angles to AC.

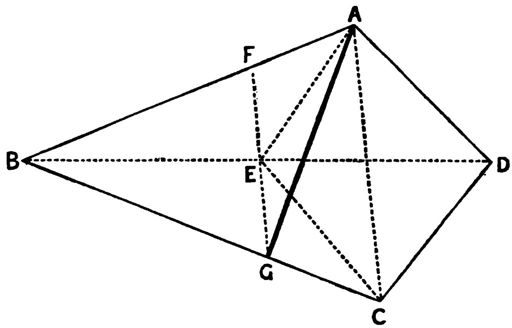
Bisect AB at right angles by EF, meeting AD in F.

With center F and radius FA describe the segment ADB.

This is the segment required.

Proof.—This problem is the 33rd proposition of Euclid's Third Book.

54. *To bisect a trapezium by a straight line drawn through one of its angular points.*



Let ABCD be the given trapezium;

A the given angular point.

Draw the diagonals AC, BD.

Bisect BD in E, and through E draw FEG parallel to AC.

Join AG.

This bisects the trapezium as required.

Proof.—The triangle AGC = the triangle AEC. (Euc. I. 37.)

To each add ADC.

∴ The fig. AGCD = the fig. AECD.

Again $\triangle$ AED = $\frac{1}{2}$ $\triangle$ ABD, because BE = ED.

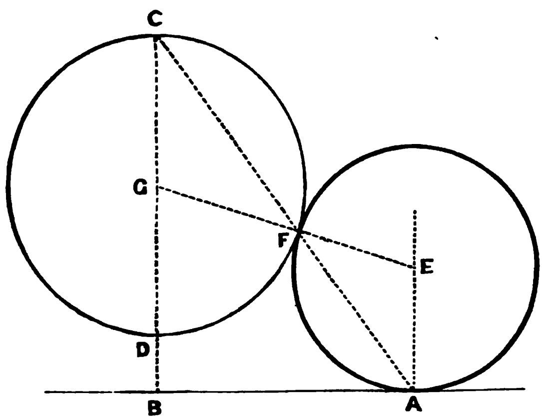
And $\triangle$ DEC = $\frac{1}{2}$ $\triangle$ BCD, because BD = ED.

$\therefore$ The fig. AECD = $\frac{1}{2}$ the given trapezium.

But the fig. AGCD = the fig. AECD.

$\therefore$ The fig. AGCD = $\frac{1}{2}$ the given trapezium.

55. *To describe a circle which shall touch a given circle, and also touch a given straight line at a given point.*



Let CFD be the given circle.

Let AB be the given straight line, and A the given point in it.

Take G the center of the given circle, and draw GB perpendicular to AB.

Let this perpendicular meet the circle in C and D.

Draw AE parallel to CB.

Join AC. Let this cut the circle in F.

Join CF, and produce it to meet AE in E.

With center E and radius EA describe a circle.

This will be the circle required.

Proof.—The angle EAF = the angle FCG by alternate angles.

= the angle GFC (Euc. I. 5).

= the angle EFA.

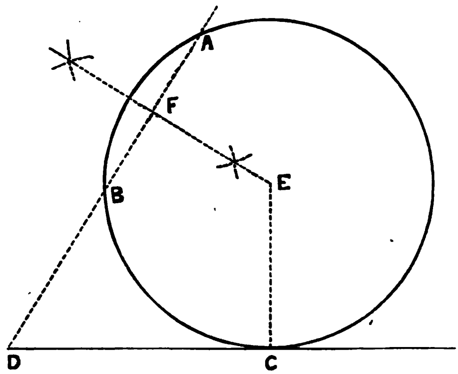
∴ EA = EF.

∴ the circle described with center E, and radius EA will pass through A and F.

It will *touch* the straight line BA at A, because the angle EAB is a right angle; and it will touch the given circle at F, because when two circles pass through a common point in the straight line joining their centers, they touch at that point.

Note.—If we had joined AD instead of AC, we should have found *another* circle satisfying the prescribed conditions; but in this case the given circle would fall *wholly within* the other circle.

56. *To describe a circle which shall pass through two given points, and touch a given straight line.*



Let CD be the given straight line, and AB the given points.

Join AB, and produce the straight line so drawn to meet CD in D.

Take DC a mean proportional between DA and DB.

Draw CE at right angles to DC.

Let the straight line bisecting AB at right angles meet CE in E.

With center E and radius EC describe a circle.

This will be the circle required.

Proof.—Since DC is a mean proportional between DA and DB, $\therefore$ DB. DA $= DC^2$.

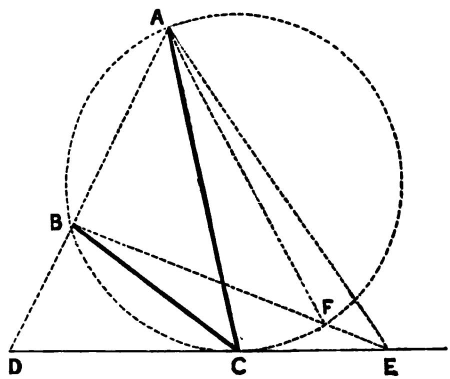
$\therefore$ DC is a tangent to the required circle (Euc. III. 37).

$\therefore$ the center is in CE.

Likewise the center is in FE (Euc. III. 1, Cor.)

$\therefore$ E is the center.

57. *To find a point in a straight line such that the angle subtended thereat by the straight line joining two given points on the same side of it is a maximum.*



Let CD be the given straight line, and A, B the given points.

Describe a circle through A, B, touching CD at C (Prob. 56).

C is the point required.

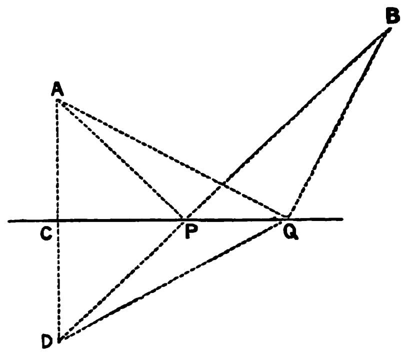
In other words, if E be *any* other point in CD, the angle ACB is greater than the angle AEB.

Proof.—The angle ACB = the angle AFB, for they are in the same segment.

And the angle AFB is greater than the angle AEB (Euc. I. 16).

∴ &c.

58. *To find a point in a straight line such that the sum of its distances from two fixed points on the same side of the line is a minimum.*



Let A, B, be the given points, and CP the given straight line.

Draw AC perpendicular to CP, and make CD = CA.

Join BD meeting CP in P.

P is the point required.

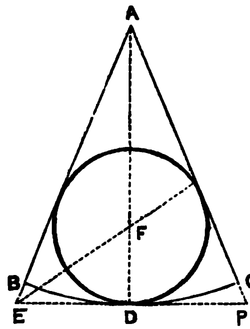
Proof.—For take Q any other point in CP, and join AQ BQ.

Then AP + BP will be less than AQ + BQ.

For by applying Euc I. 4, we find DP = AP and DQ AQ.

Hence AP + BP = DB, which is less than DQ + BQ (Euc. I. 20); and therefore AP + BP is less than AQ + BQ.

59. *To inscribe a circle in a given sector of a circle.*



Let ABC be the given sector.

Bisect the angle at A by AD, meeting the arc in D.

Draw DE at right angles to AD.

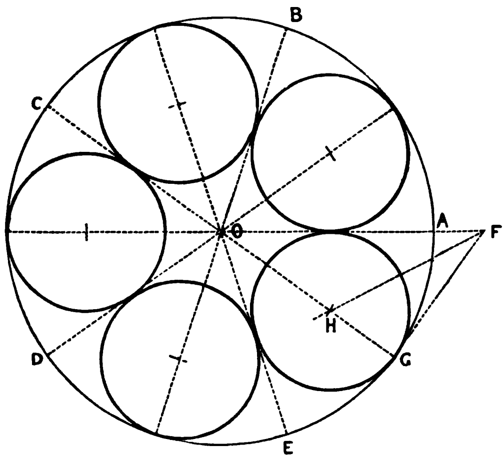
Bisect the angle AED by EF, meeting AD in F.

With center F and radius FD describe a circle.

This is the circle required.

Note.—This problem is evidently identical with that by which we inscribe a circle in a given triangle AEF.

60. *To inscribe any number (say five) of equal circles in a given circle.*



Divide the circle into five equal sectors, and bisect the angles at the center, O.

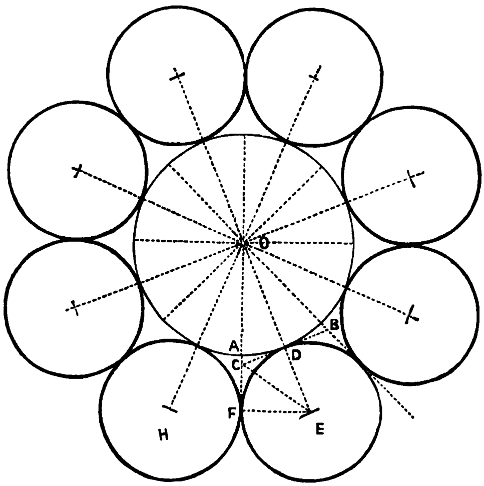
Inscribe a circle in one sector, OAE (by Prob. 59).

Let the center of this be H.

Mark off from O on the bisectors of the other sectors distances equal to OH.

These will be the centers of the remaining circles, and the radius of each will be equal to HG.

61. *To describe any number of equal circles (say eight) about a given circle, each circle to touch two others besides the given circle.*



Divide the given circle into eight equal sectors, of which let AOB be one.

Bisect the sector AOB by OD, meeting the arc in D.

Draw DC at right angles to OD.

Bisect the angle DCF by CE, meeting OD produced in E.

With center E and radius ED describe a circle.

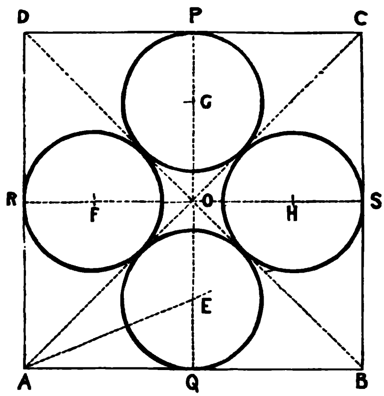
This is one of the eight circles required.

Bisect the other sectors, and mark off on these bisectors, measuring from O, distances equal to OE.

The points so found are the centers, and ED is the radius of the remaining circles.

Proof.—Apply Euc. I. 22, to prove ED = EF, &c.

62. *To inscribe four equal circles in a square—each circle to touch two others and one side of the square.*



Let ABCD be the square.

Draw the diagonals AC, BD.

The problem given therefore resolves itself into inscribing a circle in each of the four triangles AOB, BOC, COD, DOA.

This may be readily effected thus—

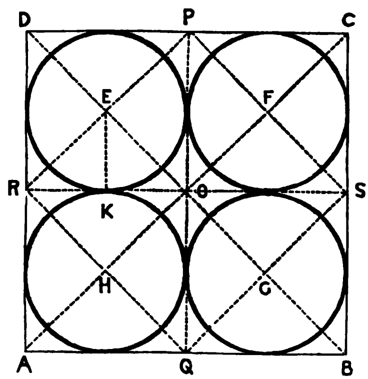
Through O draw PQ, RS, parallel to the sides of the given square.

Let AE, bisecting the angle OAB, meet PQ in E.

Mark off OF, OG, OH, each equal to OE.

E, F, G, H, are the centers, and the EQ the radius of the required circles.

63. *To inscribe four circles in a square, each circle to touch two others and two sides of the square.*



Let ABCD be the given square.

Draw the diagonals AC and BD intersecting in O.

Draw POQ parallel to CB, and ROS parallel to CD.

Join PS, SQ, QR, RP.

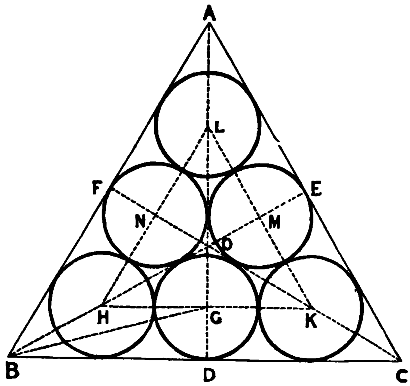
Let these meet the diagonals in G, H, E, F.

These are the centers of the given circles.

Draw EK perpendicular to OR.

This is the radius of each of them.

64. *To inscribe six circles in an equilateral triangle.*



Let ABC be the given equilateral triangle.

Let D, E, F be the mid points of the sides.

Join AD, BE, CF.

Bisect the angle CBE by BG, meeting AD in G.

Through G draw HGK parallel to BC.

Let this meet BE, CF in H, K respectively.

Draw KML parallel to AC, meeting BE in M and AD in L.

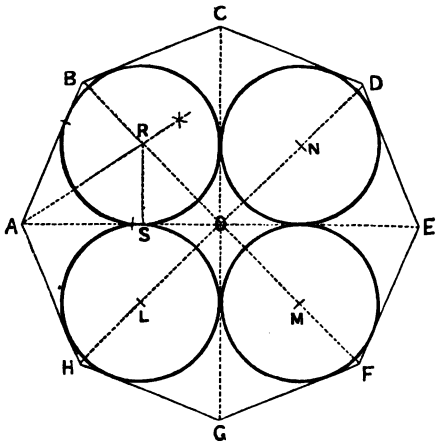
Join HL, and let this cut CF in N.

G, H, K, L, M, N are the centers of the six circles required, and the radius of each is equal to OD.

Note.—The circles whose centers are G, M, N are the inscribed circles of the triangles AOB, BOC, COA.

The circles whose centers are H, K, L are circles each having a given radius GD, and touching two intersecting straight lines (see Prob. 32).

65. *To describe four equal circles in a regular octagon.*



Let ABCDEFGH be the given octagon.

Join each pair of opposite angular points.

Let the straight lines so drawn intersect in O.

Bisect OAB by AR, meeting OB in R.

Mark off OL, OM, ON as in the figure, each equal to OR.

Then R, L, M, N are the centers of the four circles.

Draw RS perpendicular to OA.

The radius of each of the circles = RS.

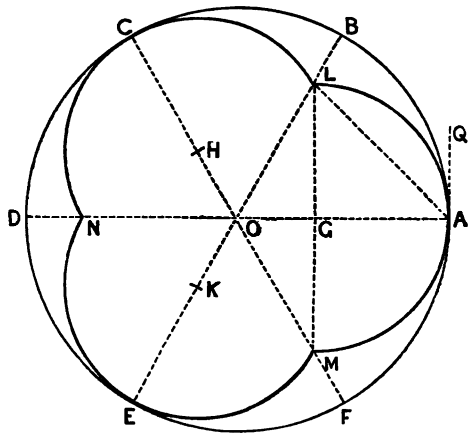
Proof.—Since the circle inscribed in OABC is to touch AB and BC, its center must lie in OB, which bisects the angle at B.

Similarly since it is to touch OA and AB, it must lie in AR.

$\therefore$ R is the center of that circle.

$\therefore$ &c.

66. THE TREFOIL.—*To inscribe three equal semicircles in a given circle, such that the three diameters form an equilateral triangle.*



Let ABC be the circle.

Divide this into six equal sectors, as in the figure.

Draw AQ at right angles to OA.

Bisect the angle OAQ by AL, meeting OB in L.

Draw LG perpendicular to OA, and produce it to meet OF in M.

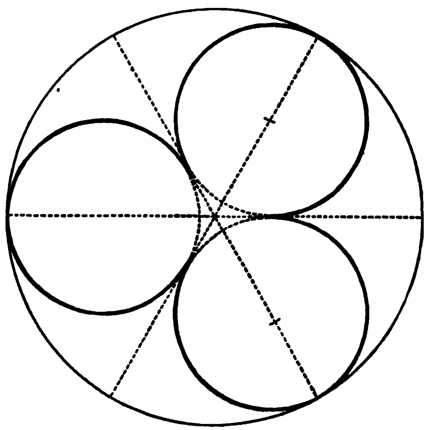
Make OH, OK each equal to OG.

G, H, K, are the centers of the three semicircles required, and the radius of each = LG.

Proof.—Since LGA is a right angle, and GAL half a right-angle, ALG is also half a right angle. Hence AG = LG. Again LG = GM. (Euc. I. 26.) Whence the proof is obvious.

Def.—A cusp is a point at which two curves meet a common tangent and stop at that point.

67. THE TREFOIL.—*To inscribe a trefoil in a circle, such that the arcs of the trefoil form cusps where they meet.*

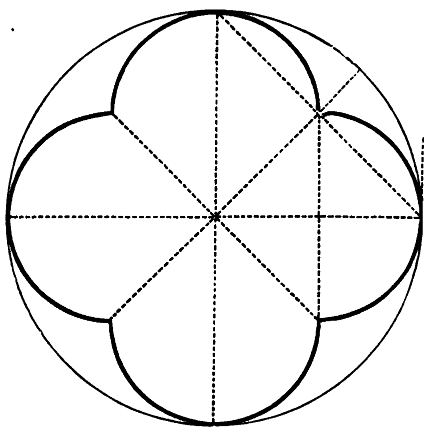


This problem differs very slightly from Problem 60. We inscribe three equal circles in the given circle, but do not produce (towards the center thereof) the arcs of the in-

scribed circles beyond their points of contact with the radii of the sectors.

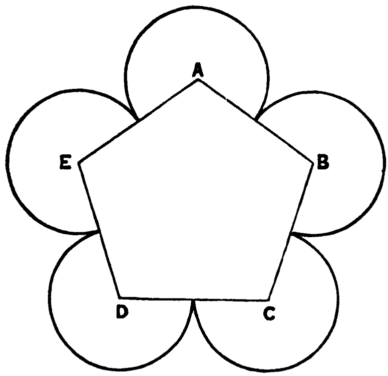
The same method holds for a cuspidate quatrefoil, cinquefoil, &c.

68. *To inscribe a quatrefoil of four equal semicircles in a given circle, so that their diameters form a square.*



The method of this problem is the same as that of Problem 64. It holds also for cinquefoils, &c., the circle being always divided into twice as many equal sections as there are semicircles to be inscribed.

69. *To construct a foiled figure about a given regular polygon. Say a cuspidate cinquefoil about a regular pentagon.*

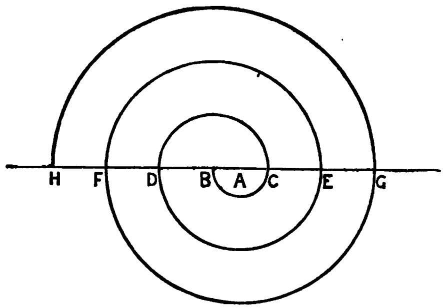


Let ABCDE be the given regular pentagon. Take the angular points of the figures as centers and with a radius equal to half the side of the pentagon describe an arc.

These will form the foiled figure required.

Note.—The figure thus constructed will be cuspidate, because when two circles pass through a common point in the line joining their centers they *touch* at that point.

70. *To construct a* **Common Spiral.**



Draw any straight line HAG.

In this take any point A.

With center A and any radius AB describe a semicircle, say *below* the line.

Let this cut the line in C.

With center B and radius BC describe a semicircle *above* the line.

Let this cut the line in D.

With center A and radius AD describe a semicircle *below* the line.

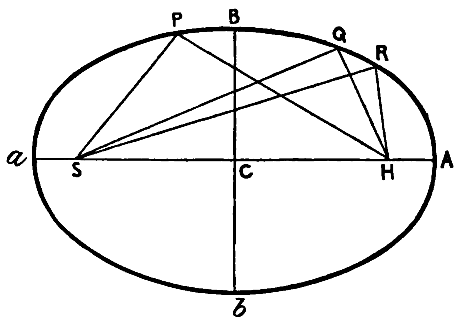
Let this cut the line in E.

Repeat this process as often as is required.

Note.—The centers are A and B *alternately*. All the circles which have A as center are *below* the line, and all which have B as center are *above* the line.

Def.—An ellipse is the locus of a point which moves in such a way that the sum of its distances from two fixed points is constant.

This accurate mathematical definition merely means that an ellipse is a curve traced out by the point of a pencil moving in such a way that its distance from one of two fixed points plus its distance from the other fixed point is always the same.



Thus, in the figure above, S and H are the fixed points, and P, Q, R being *any* points on the ellipse we have—

$$SP + HP = SQ + HQ = SR + HR.$$

S and H are called *the foci*, and the straight lines Aa, Bb the major and minor axes of the ellipse.

It is clear that since the sum of the focal distances of any point on the curve is constant, we must have this constant equal to Sa + Ha.

But Ha evidently is equal to SA.

$\therefore$ Sa + SA is the constant.

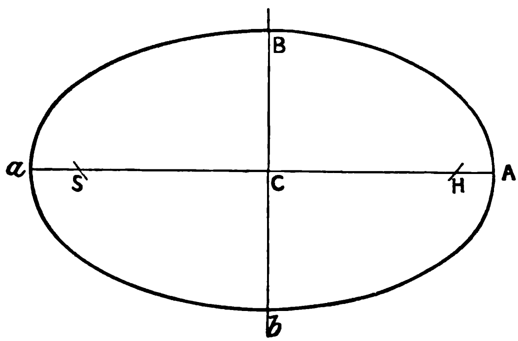
But Sa + SA = the major axis.

$\therefore$ the major axis is the constant.

In other words the sum of the focal distances of any point on an ellipse is equal to its major axis.

71. *To construct an ellipse when the major and minor axes are given. Say the major axis = $2\frac{1}{2}''$ and the minor axis $1\frac{1}{2}''$.*

(i.) *By mechanical means.*



Draw Aa, Bb intersecting at right angles in C.

Measure off CA, Ca, each equal to the semi-major axis; and CB, Cb, each equal to the semi-minor axis.

With center B and radius AC describe an arc.

Let this cut the major axis in H and S.

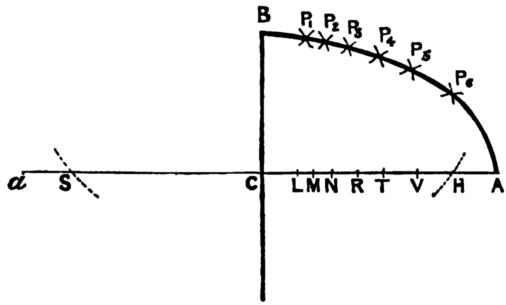
These are the foci.

Fix three pins at H, B, and S respectively.

Tie a piece of thread tightly round them so as to form the triangle SBH.

Remove the pin at B, and, keeping the string tight with the point of a pencil, trace out the ellipse.

(ii.) *By the method of intersecting arcs.*



Draw the axes at right angles to each other, and find the foci as before.

Take any points L, M, N, ... in CA.

With center S and radii aL, aM, aN ...... describe arcs, and with centre H and radii AL, AM, AN, ... describe arcs.

Let these cut the former arcs in P_1 P_2 P_3 ......

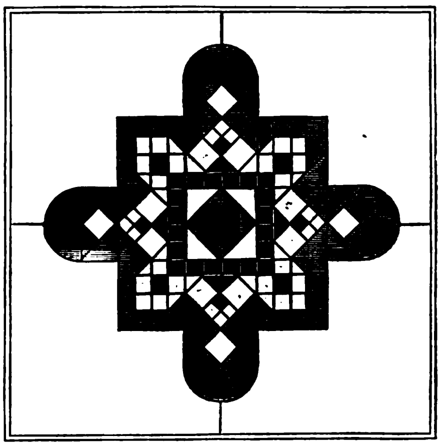
These are points on the ellipse.

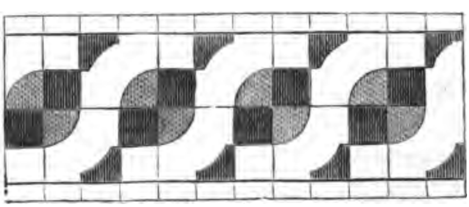
We may find as many points as we choose by this method.

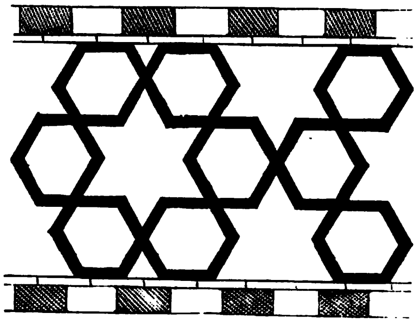
All of these must be carefully drawn by hand or with the aid of "French curves."

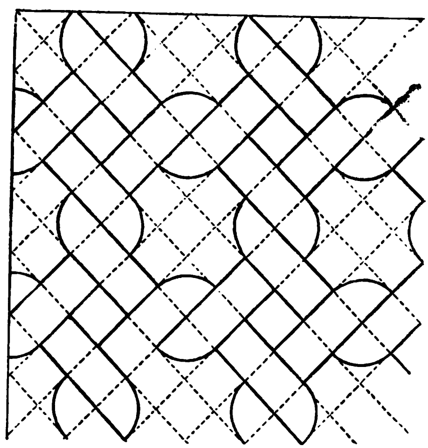
G

72. The student should now be able to combine the figures he has learned to draw, so as to form *geometrical patterns.* We subjoin one or two patterns for tesselated pavements as exercises on this part of the subject. He should draw them not only on the same scale as that given, but (say) twice as large also, or in any other proportion which may appear suitable.









EXAMPLES.

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1. Draw straight lines the lengths of which are $5.27''$, $3.28''$, $9.33''$, $3\frac{4}{7}''$, $2.\dot{4}''$, $1\frac{13}{96}''$, $\frac{7}{18}''$.

2. Describe a triangle the sides of which shall be $2\frac{2}{7}''$, $3.\dot{4}''$, $3.36''$. Find the area of this triangle in square inches.

3. The distance between two places is known to be 13 miles, and measures on a map $2\frac{1}{4}''$. Draw a scale of leagues and miles to suit the map, showing ten leagues.

Draw a comparative scale of yards by which distances of one thousand yards may be measured.

4. Two upright poles are $30'.3''$ apart, their heights are $9'.7''$ and $23'.9''$. Determine the distance apart of their tops. Scale $\frac{1}{8}''$ to $1'$.

5. Divide a line $7''$ long into 6 equal parts. On each part as diameter describe an arc of $.85''$ radius, placing the arcs alternately above and below the line so as to form a continuous curve.

6. Draw a plain scale of miles and furlongs (long enough to measure 6 miles) in which $2\frac{1}{2}$ miles is represented by $2.35''$. What is the representative fraction?

7. A length of 8ft. 9in. measures $2\frac{3}{4}''$ on a plan. Draw a diagonal scale of feet and inches to suit the plan. Show 20 feet. What is the representative fraction?

8. Draw a plain scale of 665 paces to the inch, 100 paces being the least and 5000 paces the largest dimension shown. Mark the representative fraction, assuming a pace to measure 32″.

9. A given scale is one of Russian versts, and 1 verst = .6628 English miles. Draw a comparative scales of English miles showing furlongs diagonally. What is the representative fraction?

10. The distance between two towns is 19 English miles and measures on a map 2.7″. Draw (*a*) a scale by which single miles can be measured. Show 40 miles. (*b*) A comparative scale to show 10 Austrian miles, if one Austrian mile = 3.3312 English miles.

11. Draw a diagonal scale to read leagues, miles, furlongs. Represent one league by an inch. Show 6 leagues. What is the representative fraction?

12. Make an angle of 60° without using the protractor. Divide it by trial into 3 equal parts. Bisect one of these parts, so obtaining an angle of 10°.

13. Make angles of 70°, 115°, 75°, 135°, 150°, without using the protractor.

14. Make an angle of 15° and bisect it.

15. Draw 2 straight lines AB, CD, each 3 inches long; such that C is 1″ from A, and D $\frac{3}{4}''$ from B. Draw a straight line which would bisect the angle formed by the two given straight lines if they were produced.

16. Make a triangle whose sides are 2″, $2\frac{1}{7}''$, $2\frac{4}{9}''$, and (i) inscribe a circle in it; (ii) describe a circle about it. What are the radii of these circles?

17. Describe a circle of 2.35″ radius. Inscribe a regular pentagon in it without using the protractor. Reduce the pentagon to a triangle, and the triangle to an equal octagon.

18. Draw a straight line $2\frac{13}{17}$″ long. On this describe a square. Inscribe a regular octagon in the square.

19. Inscribe 4 equal circles in the octagon of the last question.

20. Draw a 3rd proportional to two straight lines whose lengths are $1\frac{1}{3}$″ and $1\frac{1}{7}$″. What is its length?

21. Draw a mean proportional between the same 2 lines. What is its length?

22. Draw a circle of 2″ radius. Take a point 3 inches from the centre, and from it draw 2 tangents to the circle.

23. Describe 3 circles of radii 1″, 2″, $2\frac{1}{2}$″, touching each other.

24. Draw a straight line 4.37″ long. Divide it into 5 equal parts, and through the points of division draw parallels .65″ apart.

25. ABC is a triangle, and D is a point in AB such that AD = $\frac{2}{7}$ AB. Draw a straight line through D, which shall bisect the triangle.

26. Draw a straight line 3.54″ long and divide it in extreme and mean ratio.

27. With a radius of 2.5″ describe a quadrant of a circle and in it inscribe a circle. What is the radius of this circle?

28. On a straight line 2″ long describe isosceles triangles having vertical angles 100°, 120°, 130°.

29. Make a square equal to the sum of the three triangles described in the last question.

30. Construct a triangle having its sides 3.25″ 2″ and 2.5″. Divide it into four equal parts by straight lines drawn parallel to the shortest side.

31. Describe a cuspidate cinque foil, the arcs having a radius of .5″.

32. With a radius of 1.25″ describe a circle ABC, and from a point P distant 2.75″ from the center draw PBC to cut the circle so that the chord BC=2″.

33. Construct a square equal in area to the sum of three squares whose sides are 1″, 1.5″, and 2″ respectively.

34. Describe a square equal in area to the difference between two squares whose sides are 1″ and 1.5″.

35. Make a square the area of which is $\frac{5}{6}$'ths of the area of another square whose diagonal is two inches. What is the length of the side of the latter square?

36. Describe a regular pentagon whose side is $1\frac{1}{2}$″ long. Inscribe a square in it.

37. Describe a circle of 1″ radius. Inscribe five circles in it and describe ten circles about it.

38. Describe a circle of 1.75″ radius and divide into five equal and concentric annuli.

39. Describe a regular octagon having each side 1″ long. Describe another one-third of the area of the former one.

40. ABC is a triangle such that AB = 2″, BC = $2\frac{1}{2}$″, and CA= $2\frac{3}{4}$″. P is a point within the triangle, and AP

$= \frac{3}{4}''$. Draw straight lines from P which shall trisect the triangle.

41. Draw straight lines whose lengths are $\sqrt{3}''$, $\sqrt{\frac{1}{3}}''$, $\sqrt{2}''$, $\frac{2''}{\sqrt{3}}$, respectively.

42. ABC is an isosceles triangle having AB = AC, and BC is 2″ long. Describe a rectangle on BC the area of which is equal to that of the triangle.

43. If a and b are the base and altitude respectively of any triangle; x and y the base and altitude of an *equal* isosceles triangle, show that y is a fourth proportional to x, a, b.

Hence construct an isosceles triangle equal to a triangle whose sides are 2″, $2\frac{1}{2}''$, 3″, the base of the isoscees triangle being $1\frac{3}{4}''$.

44. Draw two circles having their centers 3″ apart, and their radii $1\frac{1}{2}''$ and 1″ respectively. Draw the 4 common tangents to these circles.

45. Make a parallelogram having one angle 60°, one side = 1″, and the area = that of the isosceles triangle in Question 42.

46. Construct a square having its sides = $\sqrt{3}$ inches, and in it inscribe four circles, each touching two other circles and one side of the square.

47. Plot an angle of 73° with the protractor. Divide it into 4 equal parts, and in each part inscribe a circle of one inch diameter.

48. Describe a circle of 3.25″ diameter. Consider this circle to represent the face of a watch. Draw two radii indicating the position of the hands at 20 minutes past 8. (The protractor is not to be used, and arcs may be divided up by trial.)

49. Divide a straight line 3″ long into 2 parts whose ratio is 2 : $3\frac{1}{2}$. On each side of each part describe an equilateral triangle. In each of the 2 rhombi obtained inscribe a circle.

50. Reduce the smaller of these 2 rhombi to an equal triangle, and make an equilateral triangle of equal area.

51. ABC is a triangle. Find a point P in BC, such that the perpendiculars from it on AB, AC are equal. How would you solve this problem if the directions BA and CA are given, but the point A is inaccessible?

52. Describe a square each side of which is $\frac{8}{13}$″. Describe an equilateral triangle about it.

53. Describe an ellipse by the method of intersecting arcs having for its major axis a line $\sqrt{3''}$ long, and for its minor axis a line $\sqrt{2''}$ long.

54. From a point P, two inches from the center of a circle of 1.8″ radius, draw two tangents to the circle. Join Q, R, the points of contact. Find the area of the triangle PQR.

55. Draw a straight line 2″ long, and on it describe a segment of a circle containing an angle of 100°.

56. Describe a hexagon having each side 1″ long. Remove one of the six equilateral triangles composing it; reduce the remaining figure to an equal square.

57. On a straight line 1.27″ long, describe a segment of a circle containing an angle of 70°. Inscribe a square in this segment.

58. Draw AP, AQ, two straight lines containing an angle of 36°, and across them draw another straight line 2″ long, such that AP : AQ : : 2 : 3.

59. At one end of a straight line 2.86″ long, erect a perpendicular, and trisect the right angle by an accurate geometrical construction.

60. Draw a cinque foil of adjacent semicircles of $\frac{1}{2}''$ radius.

61. Describe a circle on which lie the angular points of a triangle whose sides are in the proportion 6 : 19 : 21.

62. Show how a tangent can be drawn from an external point to a given circle without using the center of the circle.

63. On a base of 2″, construct an isosceles triangle having a vertical angle 37°, and obtain another similar triangle of half the area.

64. Draw a diagonal scale of 120 feet to the inch, to measure single feet. Show 700 feet.

65. Divide a straight line four inches long in the proportion 2 : 3 : 4 : 5. Make a quadrilateral whose sides are equal to these parts respectively, and reduce the quadrilateral to a triangle of equal area.

66. AB, AC are two intersecting straight lines; P any point from which PBC is drawn making AB = AC. Describe a circle which shall touch BC, and AB, AC produced.

67. On a straight line 1.75″ long construct a regular heptagon and an equilateral triangle on the same side of the line. Reduce the space between the outside of the triangle and the inside of the heptagon to a triangle of equal area.

68. Construct (i.) a square, (ii.) an equilateral triangle having an area of 3.27″. (The solution must be strictly geometrical.)

69. ABCDEF is a regular hexagon. G is the middle point of CD, and AG = 2.25″. Draw the hexagon.

70. The sides of a triangle are 2, $2\frac{1}{4}$, $2\frac{1}{2}$ inches respectively. Trisect the triangle by straight lines drawn from the middle point of the longest side.

71. Two angles of a triangle are 30° and 40° respectively. Construct a similar triangle having a perimeter of 4″.

72. Make a triangle equal in area to that constructed by the last question, but having its angles 30°, 60°, 90°.

73. Describe a common spiral, the diameter of the smallest semicircle being $\frac{3}{4}$″.

74. Inscribe a trefoil of adjacent semicircles in a circle of 2″ radius.

75. Inscribe a cuspidate trefoil in the same circle.

76. ABC is a triangle having the angle at A = 60°, AB = 2″, and AC = 3″. P is a point within the triangle, and AP = 1″. Trisect the triangle by straight lines drawn from P.

77. Make a square, the side of which is $\sqrt{3}$″ long, and in it inscribe 4 circles, each of which touches two other

circles and (i.) one side of the square, (ii.) two sides of the square.

78. Take 2 points 3 inches apart. With these as centres and radii $1\frac{1}{2}$, $1'$ respectively describe two circles. Draw the four common tangents to these circles.

79. Describe an isosceles triangle having each of the equal sides $= 2''$ and the vertical angle half either of the base angles. What is the magnitude of the vertical angle?

80. Divide an equilateral triangle into sixteen equal parts each of which is also an equilateral triangle.

81. A triangle on a base of one inch has an area of three square inches. What is its altitude?

82. Make a scale of chains, one mile being shown, and the smallest unit being one chain. The representative fraction to be $\frac{1}{10560}$.

83. The base of a triangle is $1''$, its altitude $1\frac{1}{2}''$ and its vertical angle $30°$. Construct it.

84. The base of a triangle is $1''$, one of its sides is $1\frac{1}{2}''$, and its vertical angle is $30°$. Construct it.

85. ABC is any triangle. On the side BC construct an equal isosceles triangle.

86. Take any triangle, ABC, and from each angular point draw straight lines at right angles to the sides of the triangle which meet at that point. Place a circle round the irregular hexagon thus described. Why is it possible to do this? Explain how it is that the area of the hexagon is twice the area of the triangle.

87. Inscribe three circles in an equilateral triangle, each circle touching the two other circles and one side of the triangle.

88. Describe an equilateral triangle about a given circle.

89. Describe about and equidistant from the sides of a given triangle, a similar triangle one of whose sides is equal to a given straight line.

90. The lengths of the straight lines joining the angular points of a certain triangle with the middle points of the opposite sides are 1.5″, and 1.75″ and 2″ respectively. Conctruct the triangle.

Note.—These bisectors meet in a point, and the distance from any angular point to this point is two-thirds of the bisector.

91. Describe a circle of 1.5″ radius. Suppose this to be the face of a watch and show accurately the position of the two hands at 25 minutes past 8 (assuming any fixed point as the position of the XII.)

92. Two circles of radii 2″ and 1″ have their centers 4″ apart. Describe a circle of 1.5″ radius which shall touch both.

Suppose the radius of the third circle had been given as x. What would be the *minimum* value of x which renders the problem capable of solution?

93. ABCD is a trapezium. The angle at A $= 120°$; AB $= 2''$; AD $= 1''$; BC $= 2''$; DC $= \frac{3}{4}''$. Bisect this trapezium by a straight line drawn from A. Make a square equal to half the trapezium.

94. ABC is a right-angled triangle having the right angle at A. Construct a square (i) which shall be equal in area to the sum of the squares described on BC, CA, AB (ii) which shall be equal in area to the difference of the squares described on AB and AC.

95. Take a circle of radius 1″ and describe a regular pentagon about it.

96. Take a straight line 3.56″ long and divide it in extreme and mean ratio. What are the lengths of the 2 segments?

97. On a straight line 2″ long as base describe an isosceles triangle having each of the base angles double the vertical angle.

98. Make a rectangle on the same base, having its area half that of the triangle described by the last question.

99. Describe by the method of intersecting arcs an ellipse having its semi-minor axis = 1.54″ and its semi-major axis = 2.86″.

100. There is a pile of spherical shot on a square base, and the diameter of each shot is 8 inches. Taking the representative fraction to be $\frac{1}{16}$, and there being 5 shot in the side of the bottom layer, draw figures showing to scale (i.) the bottom layer, (ii.) the side of the pile.

EXAMINATION FOR ADMISSION TO THE ROYAL MILITARY ACADEMY, WOOLWICH.

GEOMETRICAL DRAWING. (Obligatory.)

Monday, 5th July, 1880. 10 a.m. to 1 p.m.

N.B.—*The figures are to be neatly drawn in clear fine pencil lines. If time allows, they may be inked in with Indian ink. The double accent (") signifies inches.*

ALL CONSTRUCTION LINES MUST BE SHOWN EITHER IN PENCIL OR DOTTED IN INDIAN INK.

1. On a plan 6.5" represents an English mile, or 1,760 yards.

(*a.*) Draw a plain scale of yards to suit the plan, showing 1,500 yards, and divide it to show distances of 50 yards.

(*b.*) Draw a comparative scale of Spanish yards. A Spanish yard = .927 of an English yard.

Each scale to be properly figured, and all calculations shown.

2. The hypotenuse of a right-angled triangle is 2.75", and the other two sides are to each other in the ratio of 3 to 5. Construct the triangle.

3. Draw two straight lines 2" apart and parallel to each other by construction. Take a point P between them and distant .5" from one of them. Describe a circle that shall touch the two straight lines and pass through the point P.

4. Describe a circle of 1.5″ radius, and in it inscribe three equal circles, each touching the other two and the original circle.

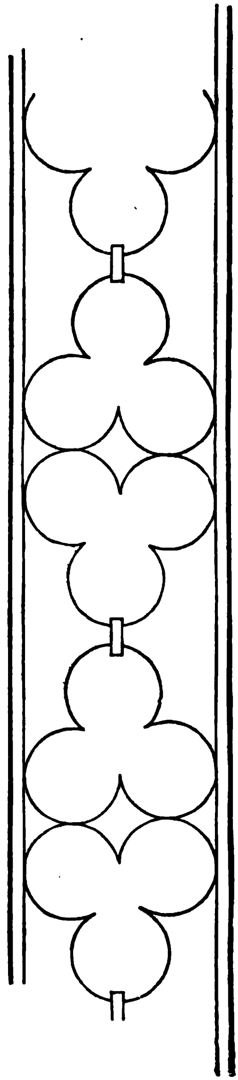
5. Construct the trapezium ABCD. Given AB = 2·2″; AD = 1.5″, BC = 2.85″, DC = 3.75″, and bisect the figure by a straight line drawn from B.

6. Draw a straight line AB, and take a point P one inch above it. Draw a curve, every point of which shall be equally distant from AB and the point P.

7. Draw the geometrical pattern shown, making it half as large again as the copy.

N.B.—This should be inked in if possible.

Q. 7.



EXAMINATION FOR ADMISSION TO THE ROYAL MILITARY ACADEMY, SANDHURST.

GEOMETRICAL DRAWING.

[*Obligatory in the case of Candidates who have not already qualified in this subject.*]

Tuesday, 6th July, 1880. 10 a.m. to 12.30 p.m.

N.B.—*The figures may be inked in if time allows, but full credit will be given for good pencilling. The double accent (″) signifies inches.*

1. The length of an ordinary pace is 30″; in "stepping short" it is 21″: draw a scale of $\frac{1}{3000}$ to show 600 ordinary paces, and also a comparative scale of "short" paces.

2. In the given figure the five points of the star are situated at the angles of a regular pentagon. Draw the figure from the dimensions attached. (N.B.—No marks will be given if the diagram is merely copied, or pricked off.)

3. Draw a straight line AA 6″ long, and take a point P 2.25″ above it: from the point P draw all the lines which make angles of 38°, 56°, and 74°, with the line AA.

4. Construct four concentric, similarly situated hexagons 0.3″ apart; the side of the largest to be 2.25″.

5. Draw two lines cutting each other at an angle of 55°, and describe all the circles of 2″ diameter which will touch both.

6. Between two concentric arcs of circles of 2.4″ and 3.7″